NOTCHES
AND
THEIR EFFECTS

CHAPTER OUTLINE

- Background
- Stress/Strain Concentrations
- S-N Approach for Notched Members
- Strain-Life Approach for Notched Members
- Applications of LEFM to FCG at Notches
- The Two-Stage Approach

BACKGROUND

- Notches cannot be avoided in many structures and machines and notch effects have been a key problem in the study of fatigue.

- Examples:
  - Thread roots and the transition between the head and the shank
  - Rivet holes in sheets
  - Welds on plates
  - Keyways on shafts

- Although notches can be very dangerous they can often be rendered harmless by suitable treatment.

BACKGROUND

- To understand the effects of notches one must consider five parameters:

    1. Concentrations of stress and of strain.
    2. Stress gradients.
    3. Mean stress effects and residual stresses.
    4. Local yielding.
    5. Development and growth of cracks.

STRESS AND STRAIN
CONCENTRATIONS AND GRADIENTS

- The degree of stress and strain concentration is a factor in the fatigue strength of notched parts.

- It is measured by the **elastic stress concentration factor**, K_t:

$$K_t = \frac{\sigma}{S} = \frac{\varepsilon}{e}$$

As long as σ/ε = constant = E

Where:

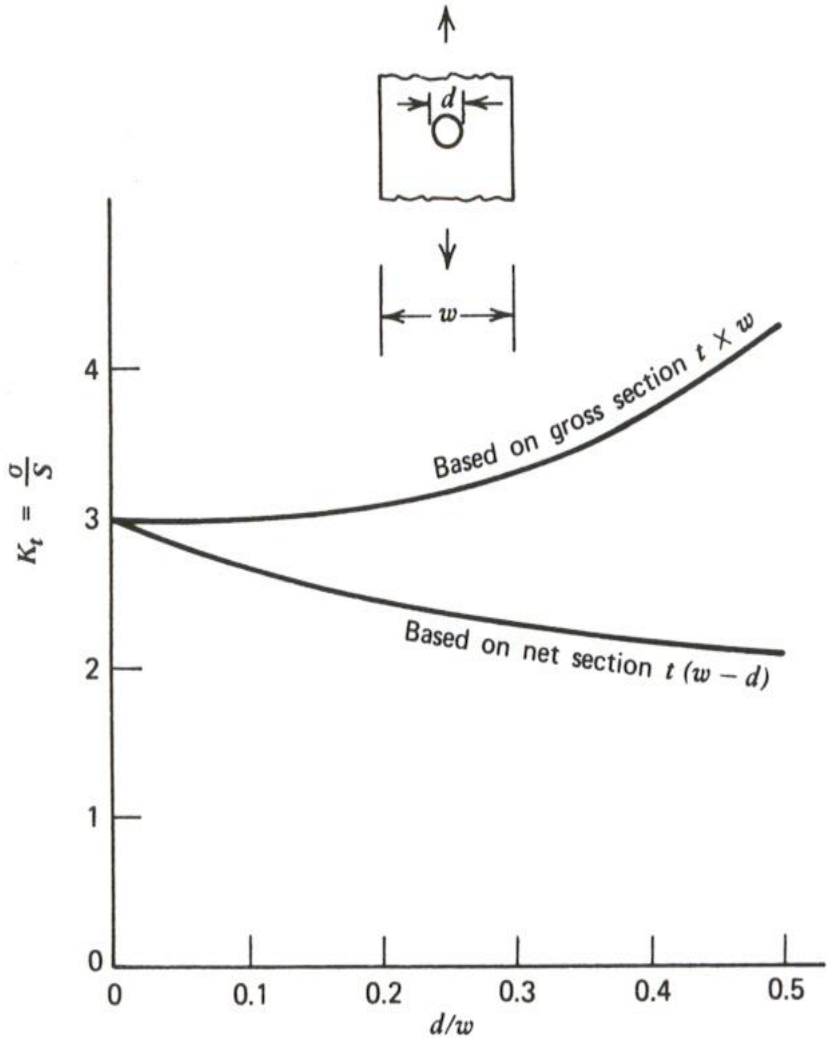
σ or ε = the maximum stress or strain at the notch

S or e = the nominal stress or strain

STRESS AND STRAIN CONCENTRATIONS AND GRADIENTS

- ***K_t*** plotted vs the ratio of hole diameter to sheet width.
  - In the upper curve the nominal stress is defined as load divided by total or **gross area** (*w* x *t*).
  - In the lower curve the nominal stress is defined as load divided by **net area**.

*In this book we use the **net area** to define the nominal stress when using stress concentration factors. However, in calculating the stress intensity factor from the nominal stress we use the gross area as if the crack did not exist, as in Ch 6.*



STRESS AND STRAIN CONCENTRATIONS AND GRADIENTS

- Figure 7.2 shows stresses near a circular hole in the center of a wide sheet in tension.

- The following equations represent the axial stress σ_y and the transverse stress σ_x:

$$\frac{\sigma_x}{S} = 1.5\left(\frac{r}{x}\right)^2 - 1.5\left(\frac{r}{x}\right)^4$$

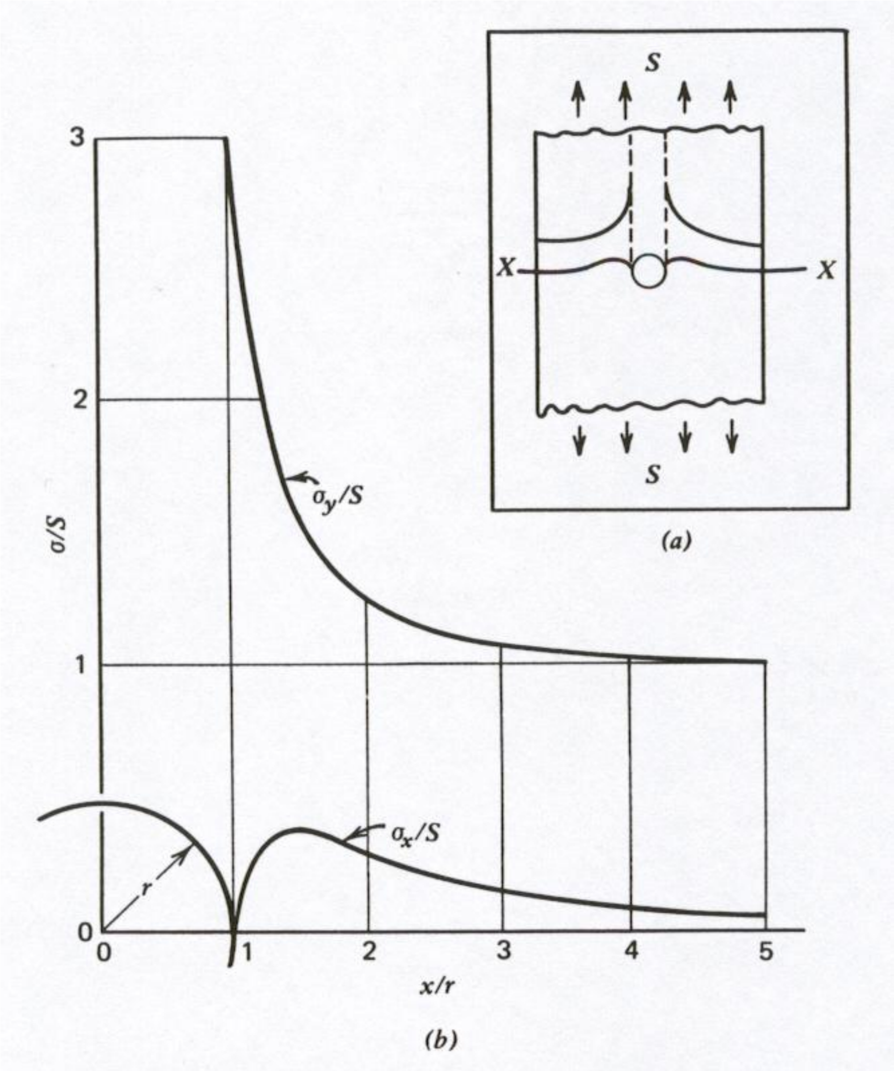
$$\frac{\sigma_y}{S} = 1 + 0.5\left(\frac{r}{x}\right)^2 + 1.5\left(\frac{r}{x}\right)^4$$

S = nominal stress = load/area

σ_y = axial stress σ_x = transverse stress

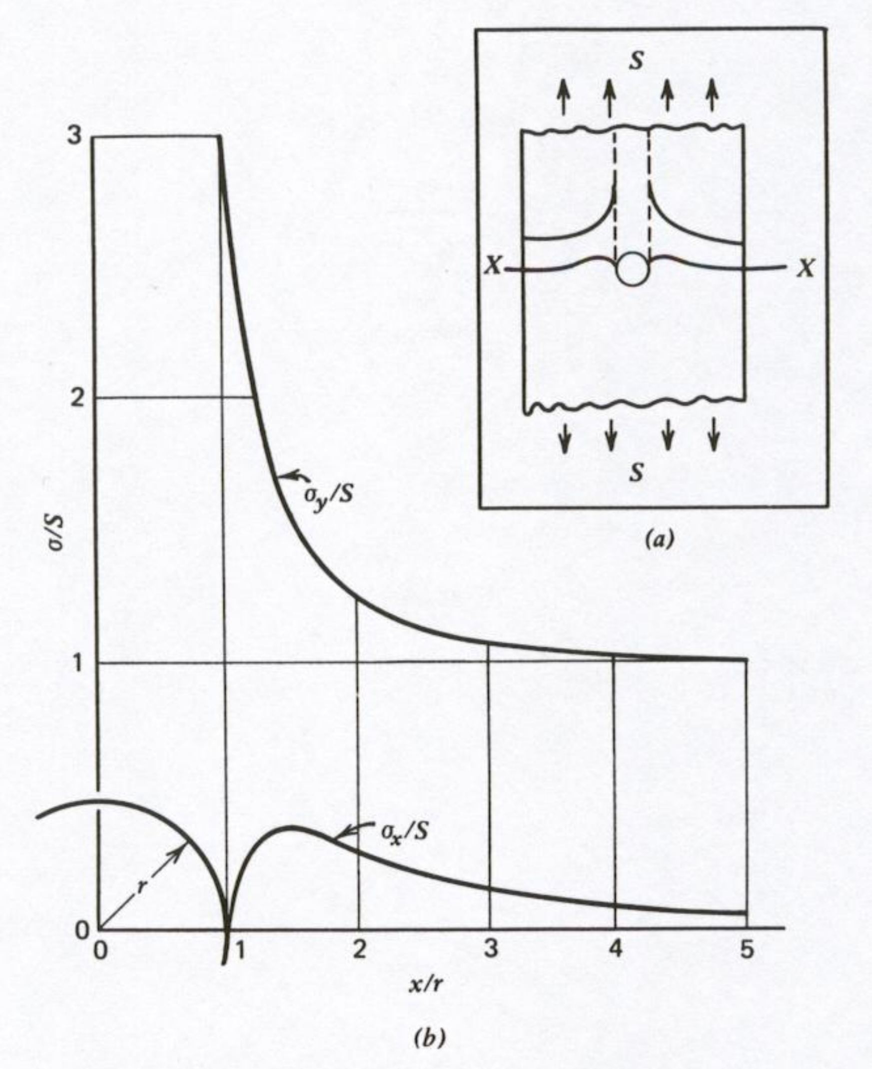
x = distance from center of hole

r = radius of hole



STRESS AND STRAIN CONCENTRATIONS AND GRADIENTS

- Values of σ_y/S and σ_x/S are plotted versus x/r.
- σ_y/S decreases quite rapidly as the distance from the edge of the hole is increased.
- K_t at the edge of the hole is 3, while at a distance of $0.25r$ the value of σ_y/S is only about 2. At a distance of $2r$ it is only 1.07.
- Rapid decrease of stress with increasing distance from the notch and existence of biaxial or triaxial states of stress at a small distance from the notch are typical of stress concentrations.

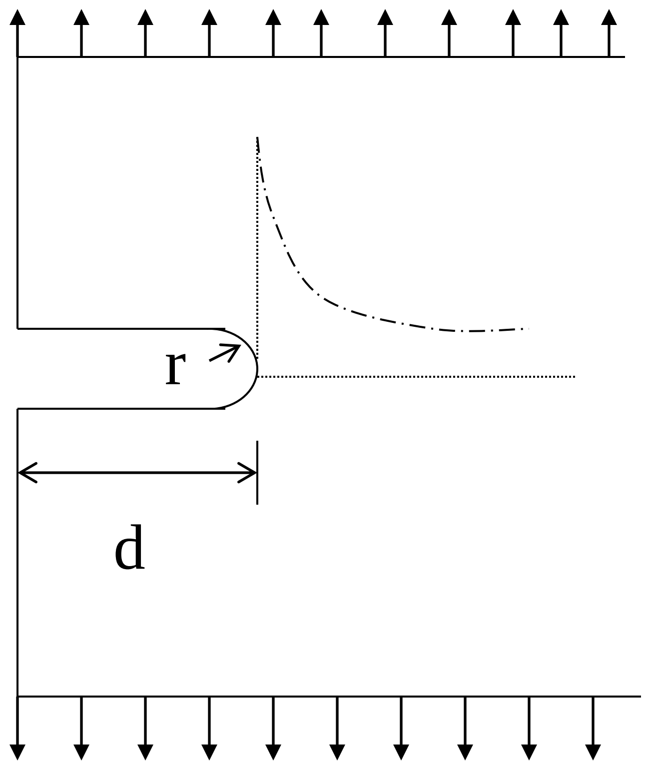


STRESS AND STRAIN CONCENTRATIONS AND GRADIENTS

- For deep narrow notches with semicircular ends a formula analogous to linear elastic fracture mechanics formulas has been given for the stress distribution:

$$\sigma_y = \sigma_{max}\left(\frac{0.5r}{0.5r + d}\right)^{1/2}$$

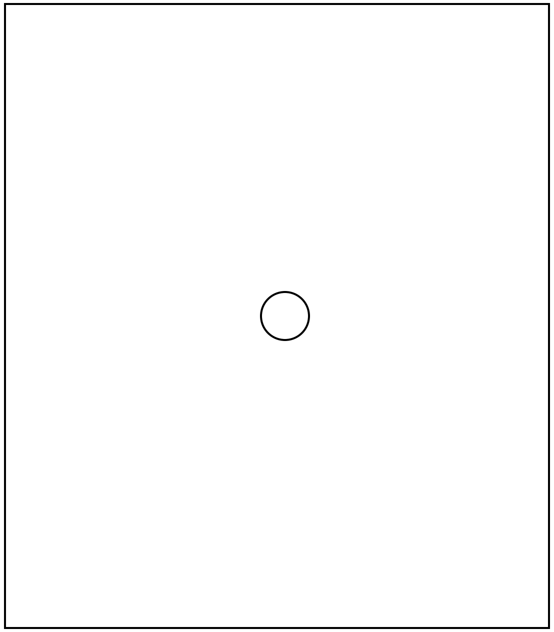
where d is the distance from the edge of the notch of radius r.



STRESS AND STRAIN CONCENTRATIONS AND GRADIENTS

- The stress concentration produced by a given notch is not a unique number as it depends on the **mode of loading**.

- For instance, for the circular hole in a wide sheet:
  - In tension: 3
  - In biaxial tension: 2



STRESS AND STRAIN CONCENTRATIONS AND GRADIENTS

- Elastic stress concentration factors are obtained from:
  - Theory of elasticity
  - Numerical solutions
  - Experimental measurements

- The most common and flexible numerical method is the finite element method. A model with relatively fine mesh in the areas of steep stress gradients is required.

- **Experimental** measurement techniques widely used include
  - Brittle coatings
  - Photoelasticity
  - Thermoelasticity
  - Strain gages.

STRESS AND STRAIN CONCENTRATIONS AND GRADIENTS

- Brittle Coating Technique:
  - A brittle coating is sprayed on the surface and allowed to dry.
  - Crack patterns developed by the loading and their relation to a calibration coating indicate regions and magnitudes of stress concentrations.

- Photoelasticity Technique:
  - A specimen with identical geometry to the actual notched part is made of a certain transparent material.
  - Changes in optical properties of the transparent material under load, measured by a polariscope, indicate stress distributions and magnitudes.

STRESS AND STRAIN CONCENTRATIONS AND GRADIENTS

- Thermoelasticity Technique:
  - Stress distribution is obtained by monitoring small temperature changes of the specimen or component subjected to cyclic loading.

- Electrical Resistance Strain Gage:
  - The most common experimental measurement technique
  - A strain gage is bonded to the surface in the region of interest.
  - Applied load causes dimensional changes of the gage resulting in changes to electrical resistance, which in turn indicates the existing strain.

STRESS AND STRAIN CONCENTRATIONS AND GRADIENTS

- Charts of stress concentration factors are available in the literature.
- Examples of such charts for:
  - Stepped shafts in
    - tension,
    - bending, and
    - torsion
  - A plate with opposite U-shaped notches in
    - tension and
    - bending
- Elastic stress concentration factors depend only on geometry (**independent of material**) and mode of loading, and that they only apply when the notch is under **linear elastic deformation condition**.

Stepped shaft in tension, bending, and torsion

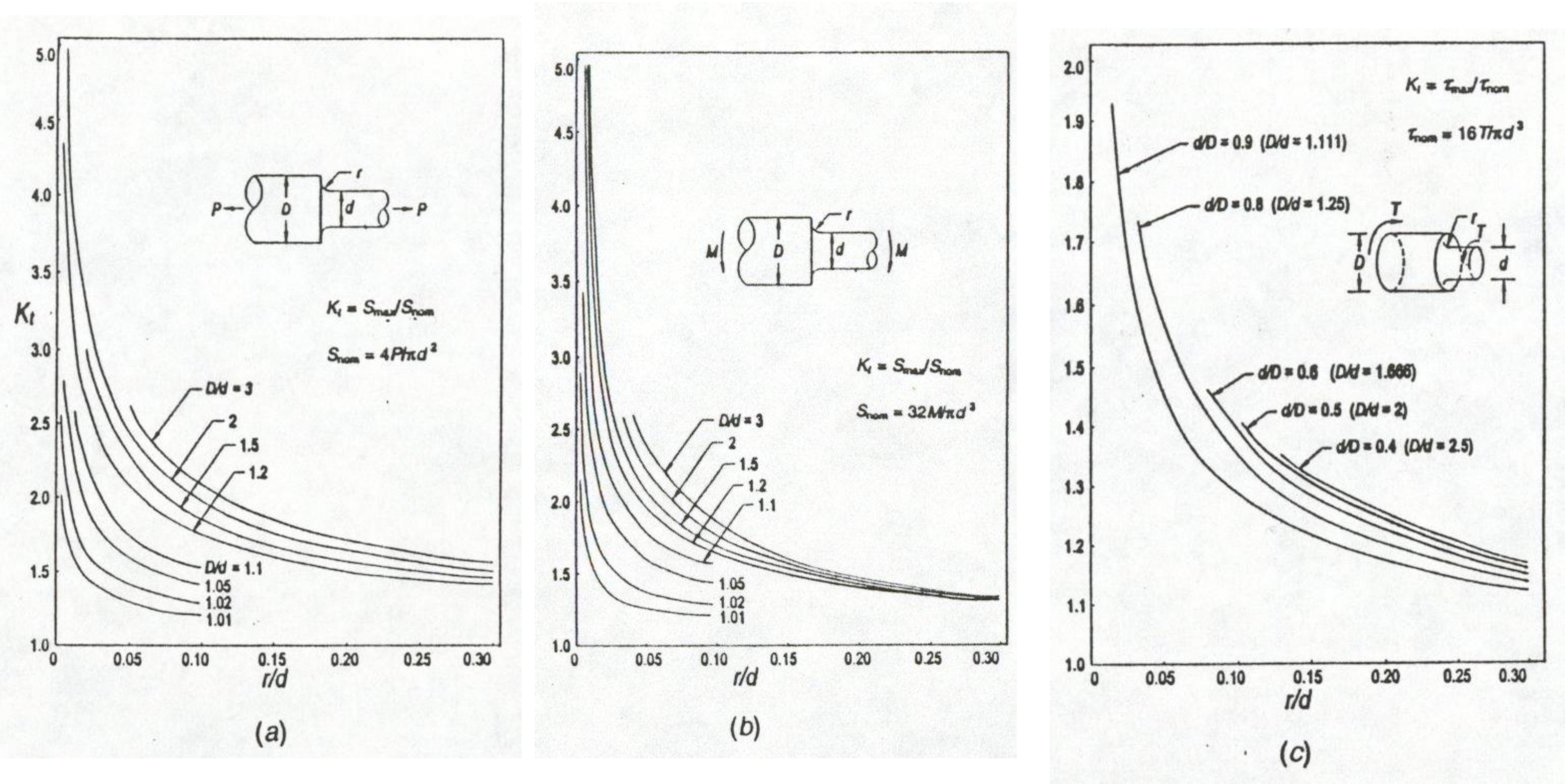
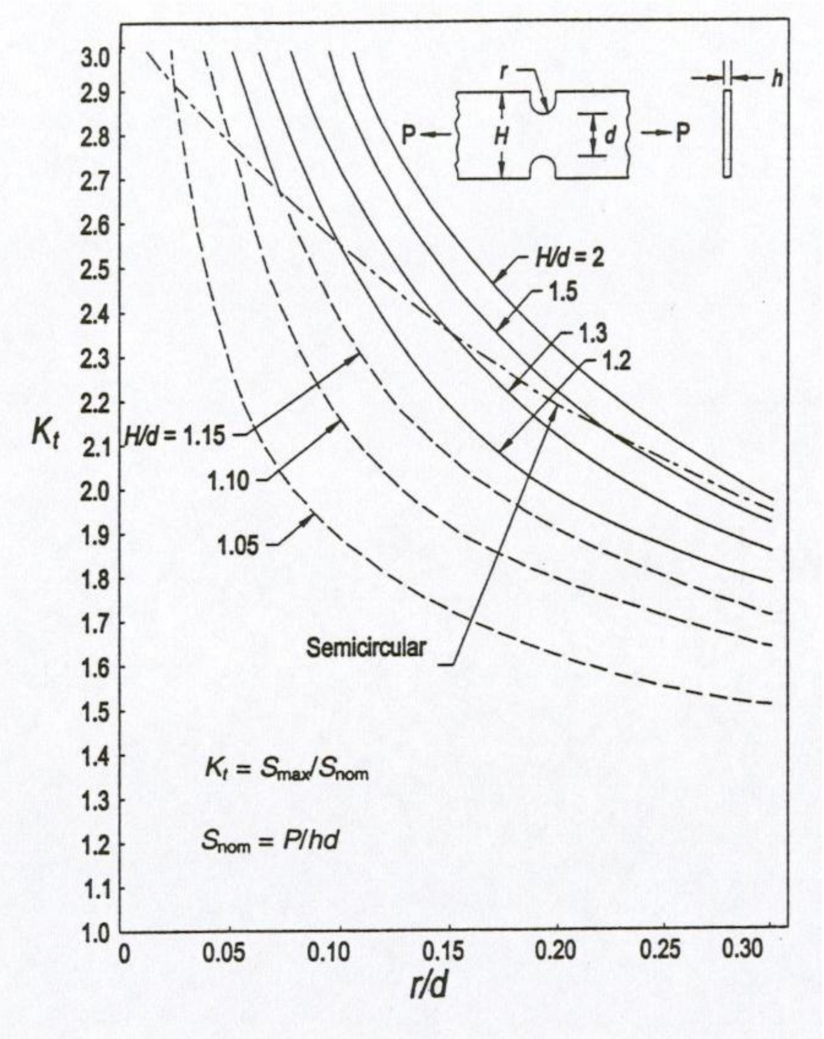
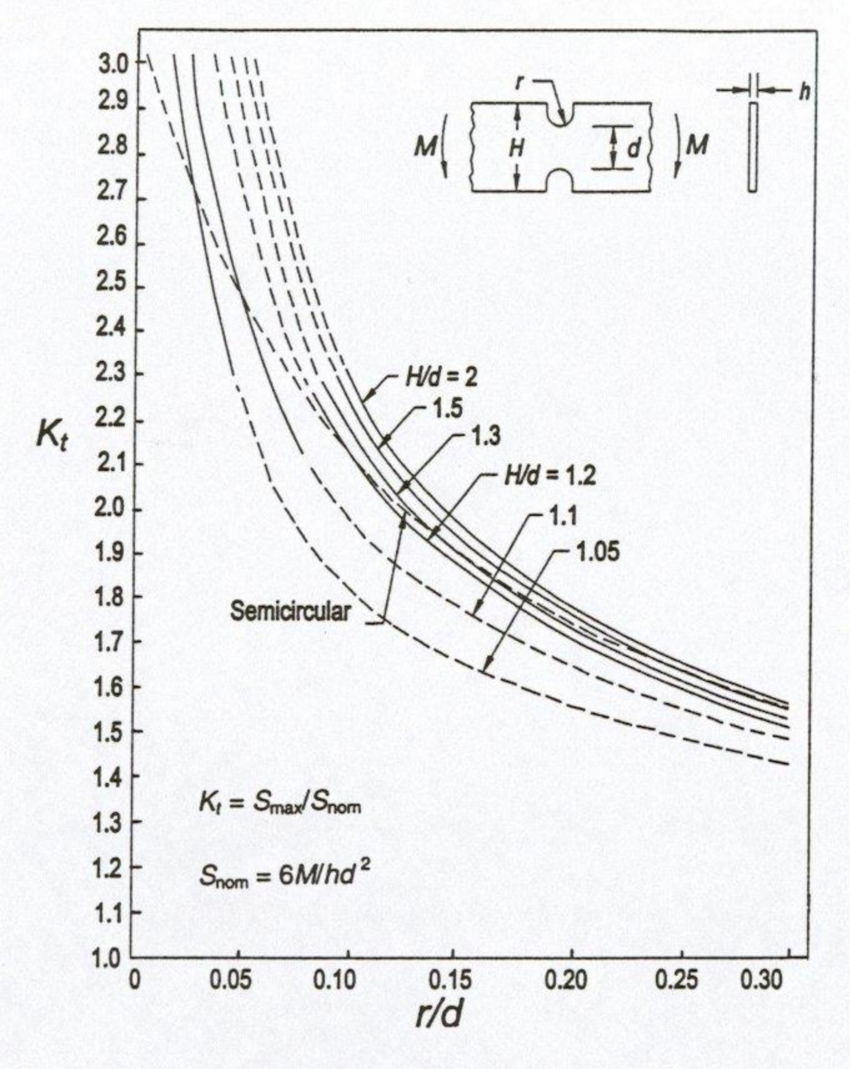


plate with opposite U-shaped notches in tension and bending





STRESS AND STRAIN CONCENTRATIONS AND GRADIENTS

- For qualitative estimates we can use an analogy between stresses or strains and liquid flow.
  - Restrictions or enlargements in a pipe produce local increases in flow velocity somewhat similar to the local increases in stresses produced by changes in cross section.
  - The designer will try to "streamline" the contours of parts as indicated in Fig. 7.5.

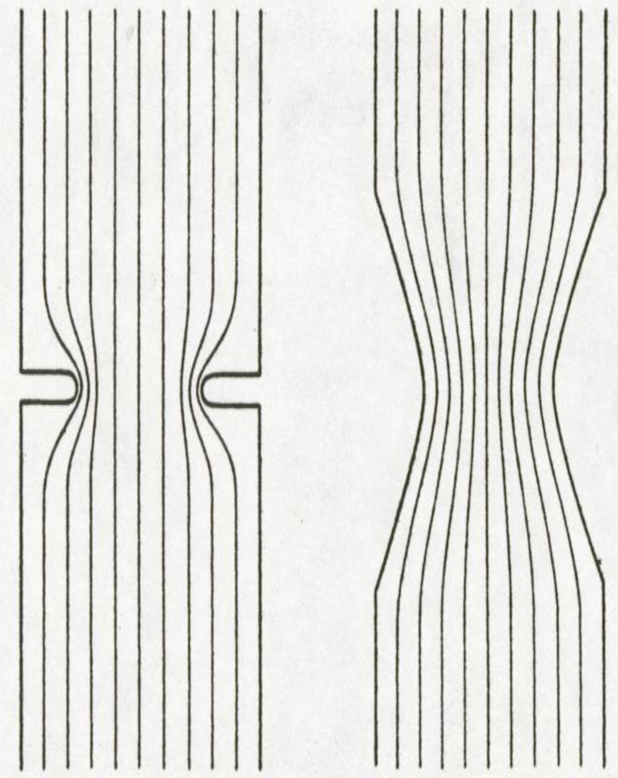
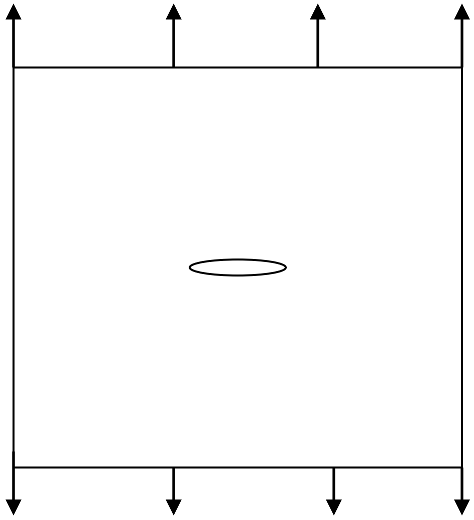
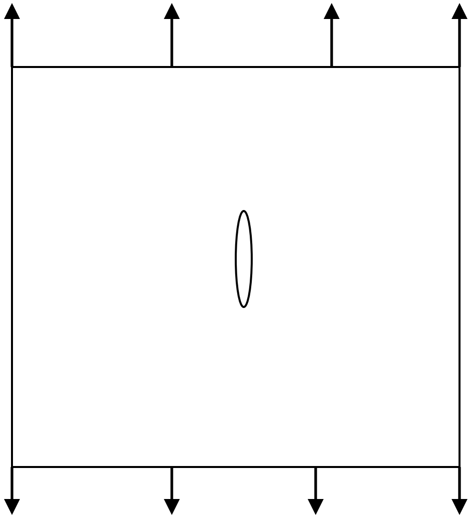


Figure 7.5 The crowding and bending of flow lines near obstructions helps to visualize the concentration of stresses and strains near notches. The large section and the small section are the same in both cases, but the transitions are different.

STRESS AND STRAIN CONCENTRATIONS AND GRADIENTS

Consider for instance an elliptic hole in a wide sheet. Placed lengthwise with the forces or flow it produces less stress concentration and less flow interference than when it is placed crosswise.



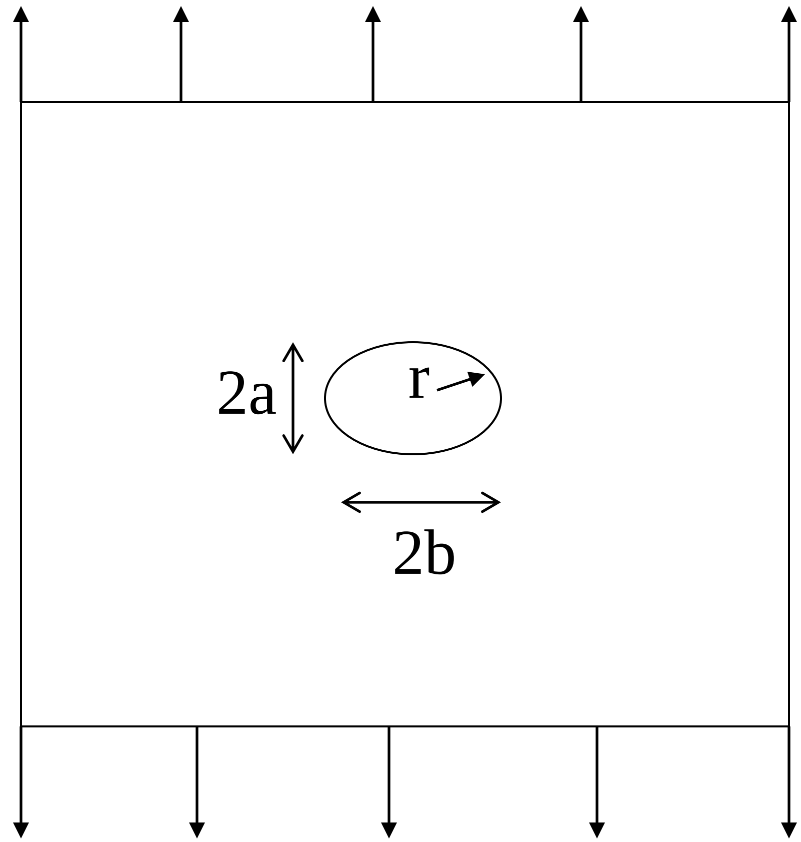


STRESS AND STRAIN CONCENTRATIONS AND GRADIENTS

- K_t produced by an **elliptic hole** with principal axes 2*a* and 2*b* is:

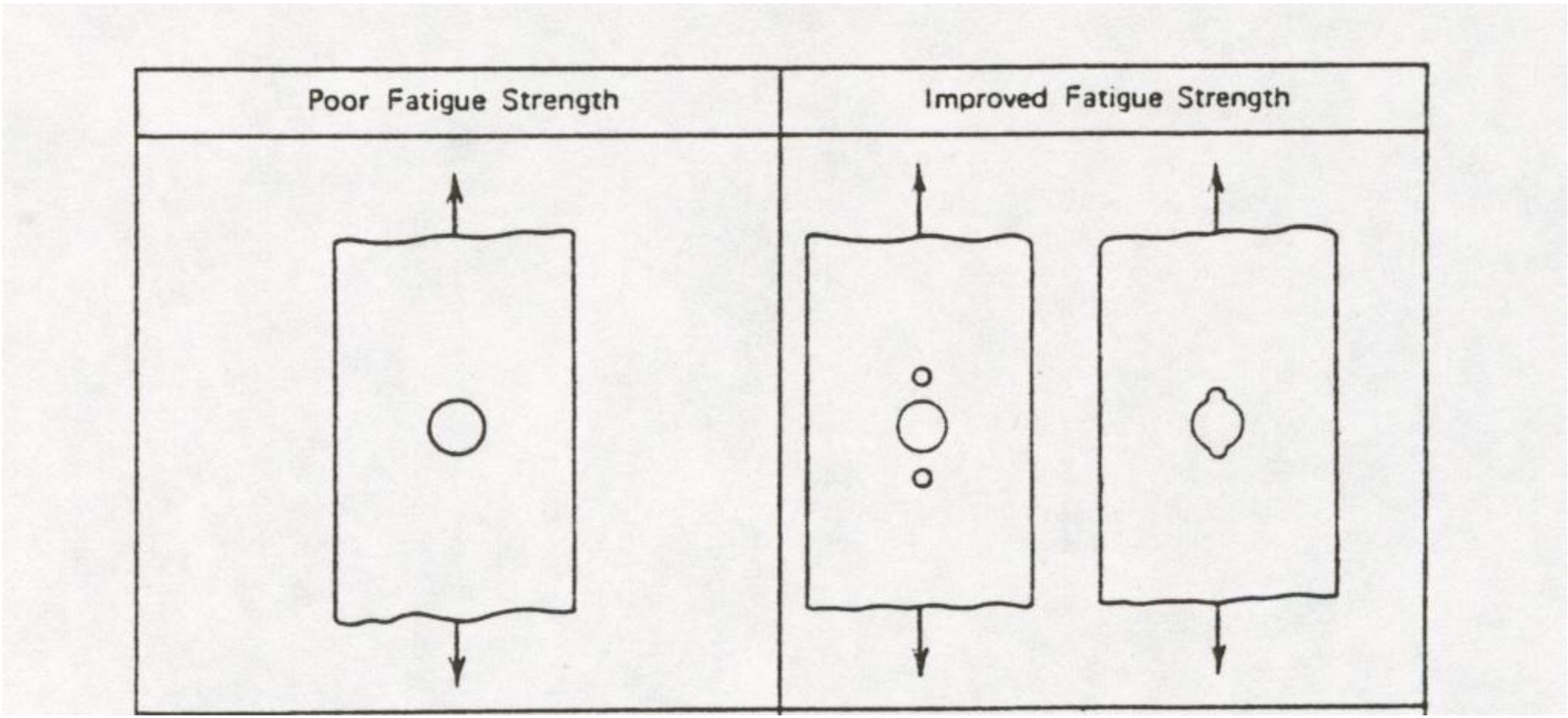
$$K_t = 1 + 2\frac{b}{a} = 1 + 2\sqrt{\frac{b}{r}}$$

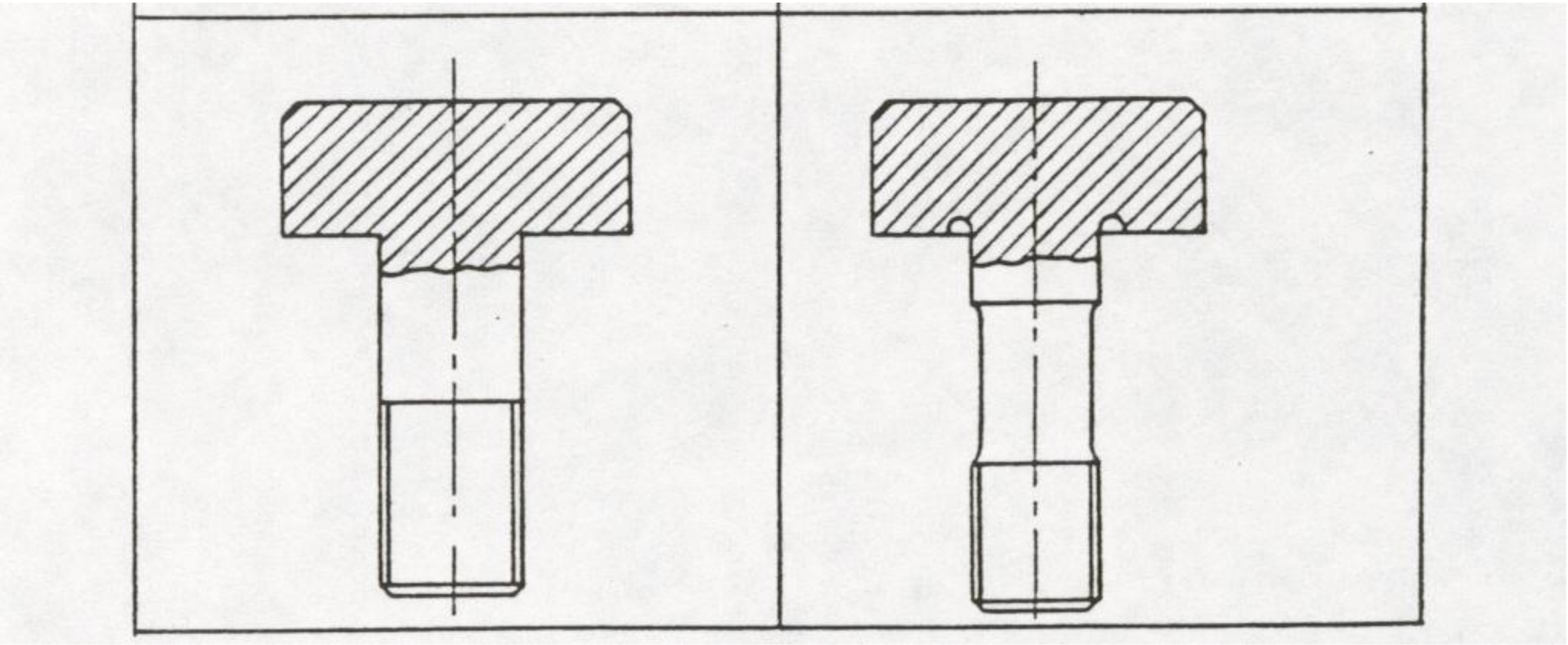
where *b* is the axis transverse to the tension force and *r* is the radius of curvature at the endpoint of *b*.

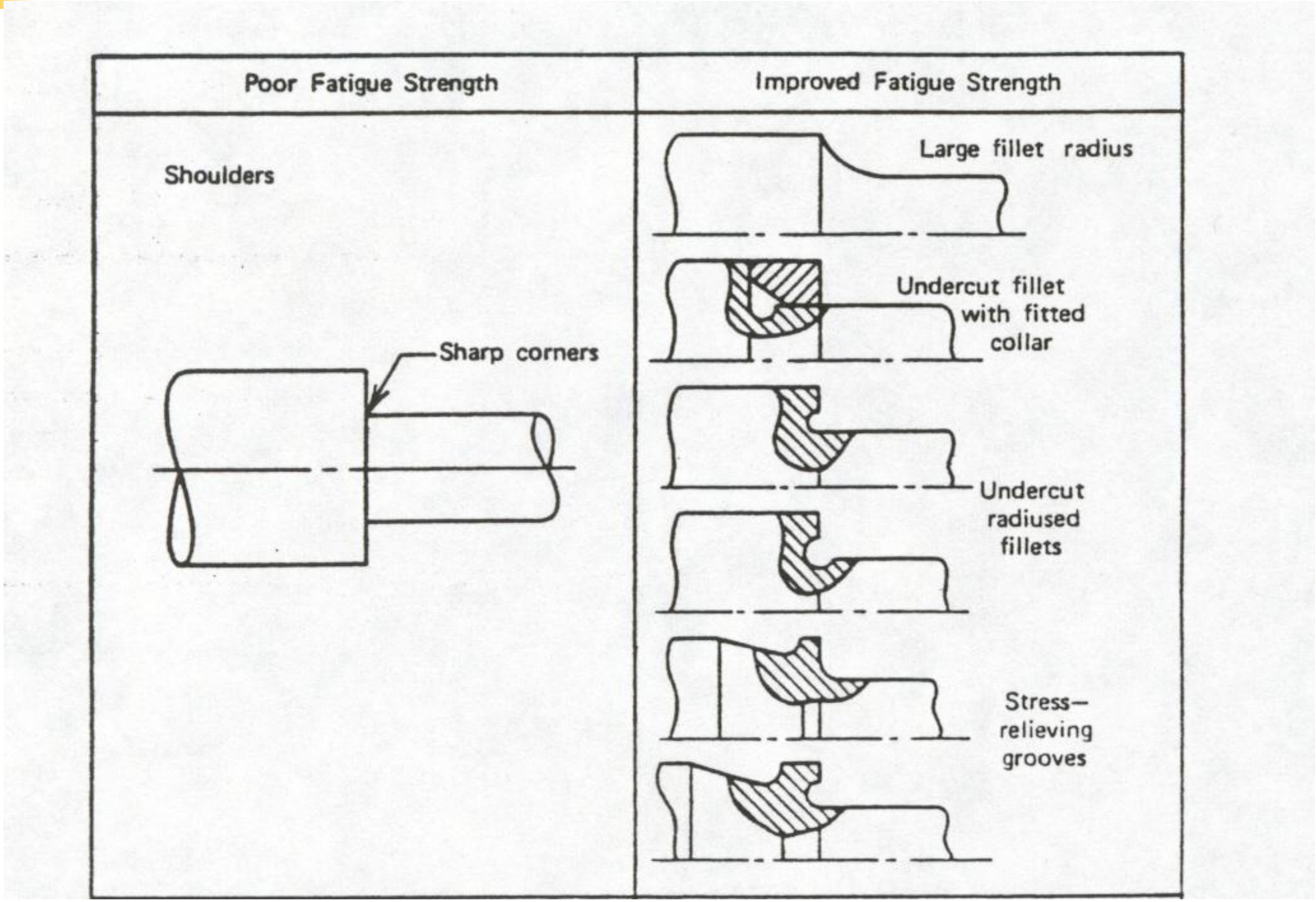


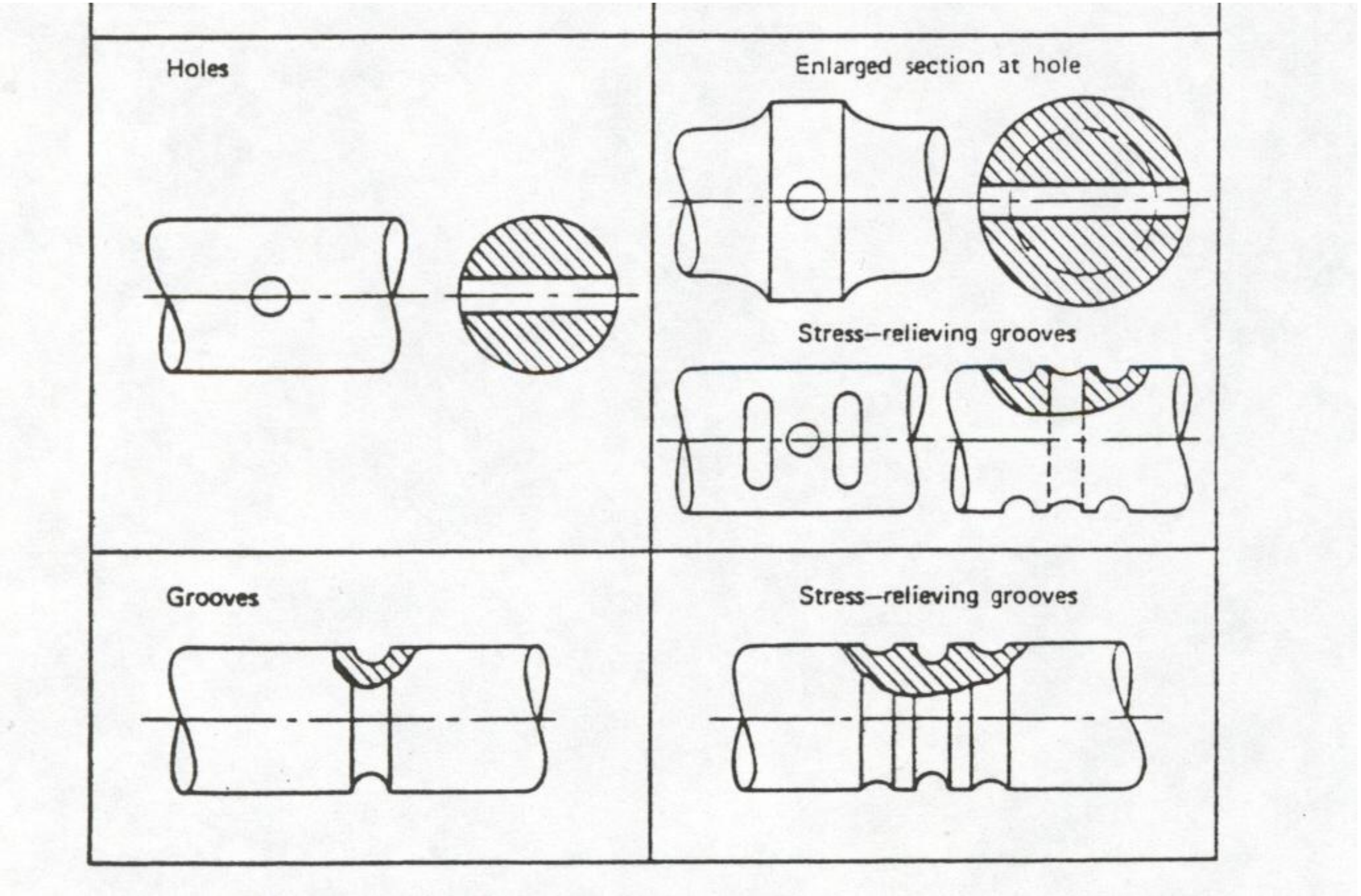
- With an ellipse 30 mm long and 10 mm wide the stress concentration is:
  - K_t = 7 if placed crosswise
  - K_t = 1.67 if placed lengthwise

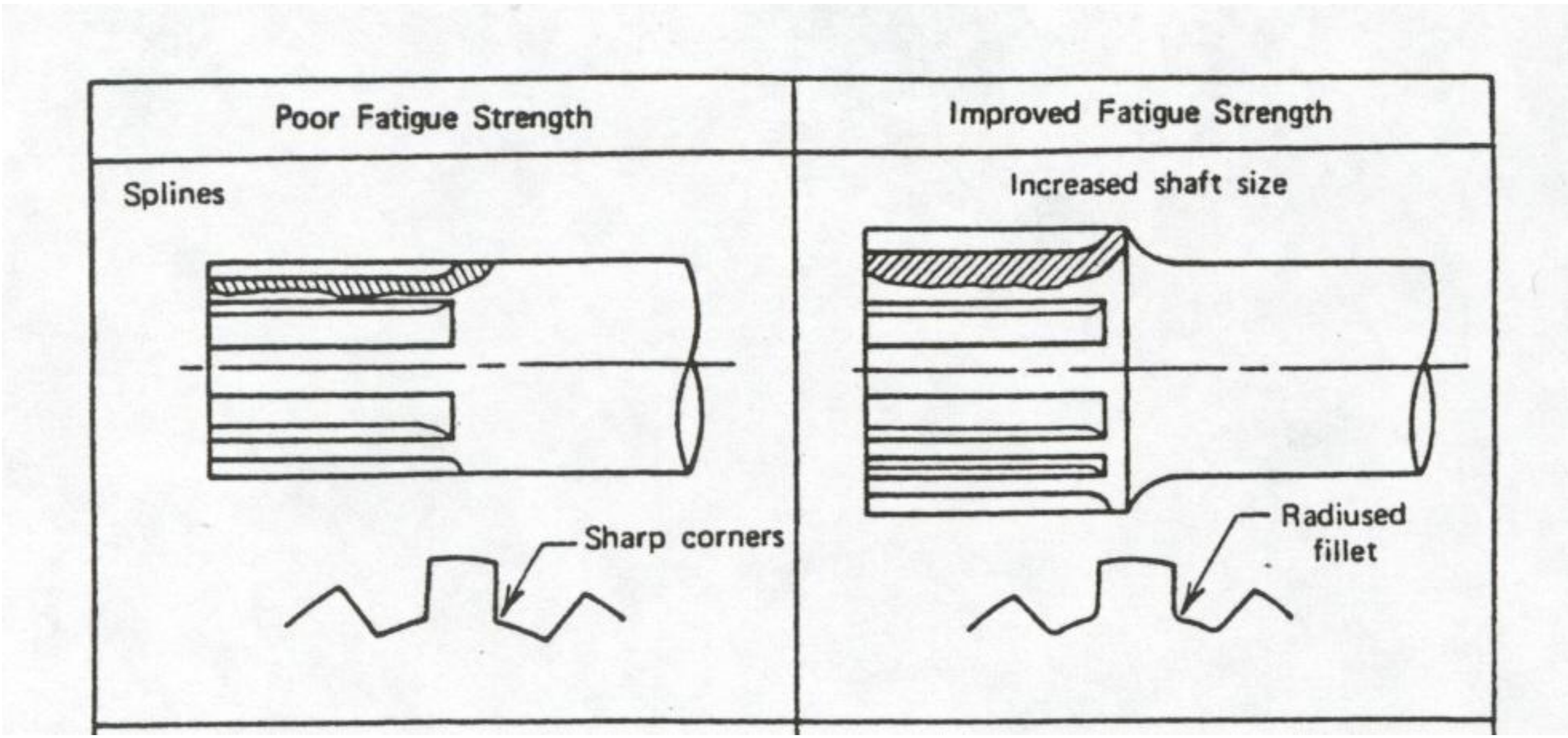
- Other examples of mitigating stress concentrations are given in Fig. 7.6.

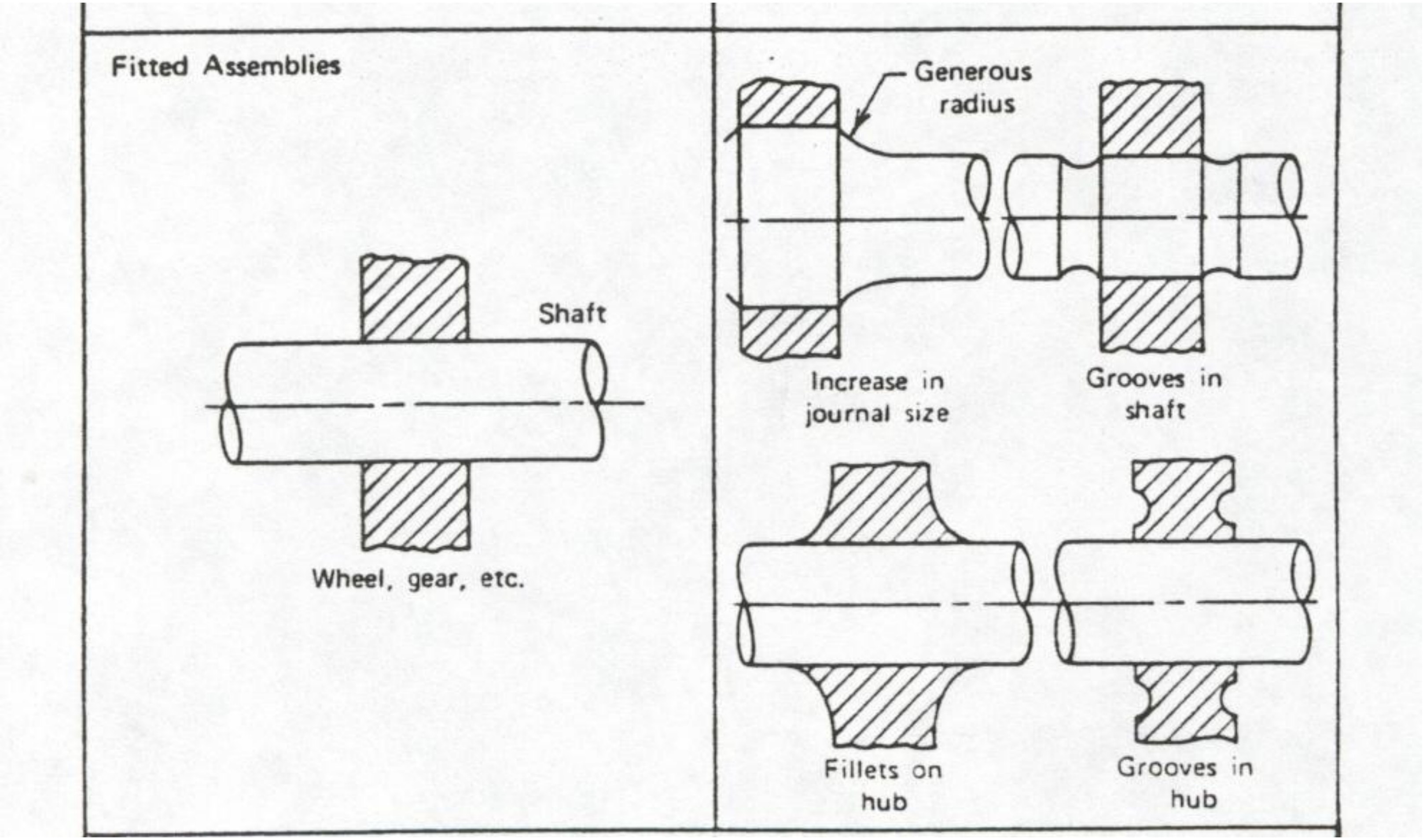

Poor Fatigue Strength
Improved Fatigue Strength

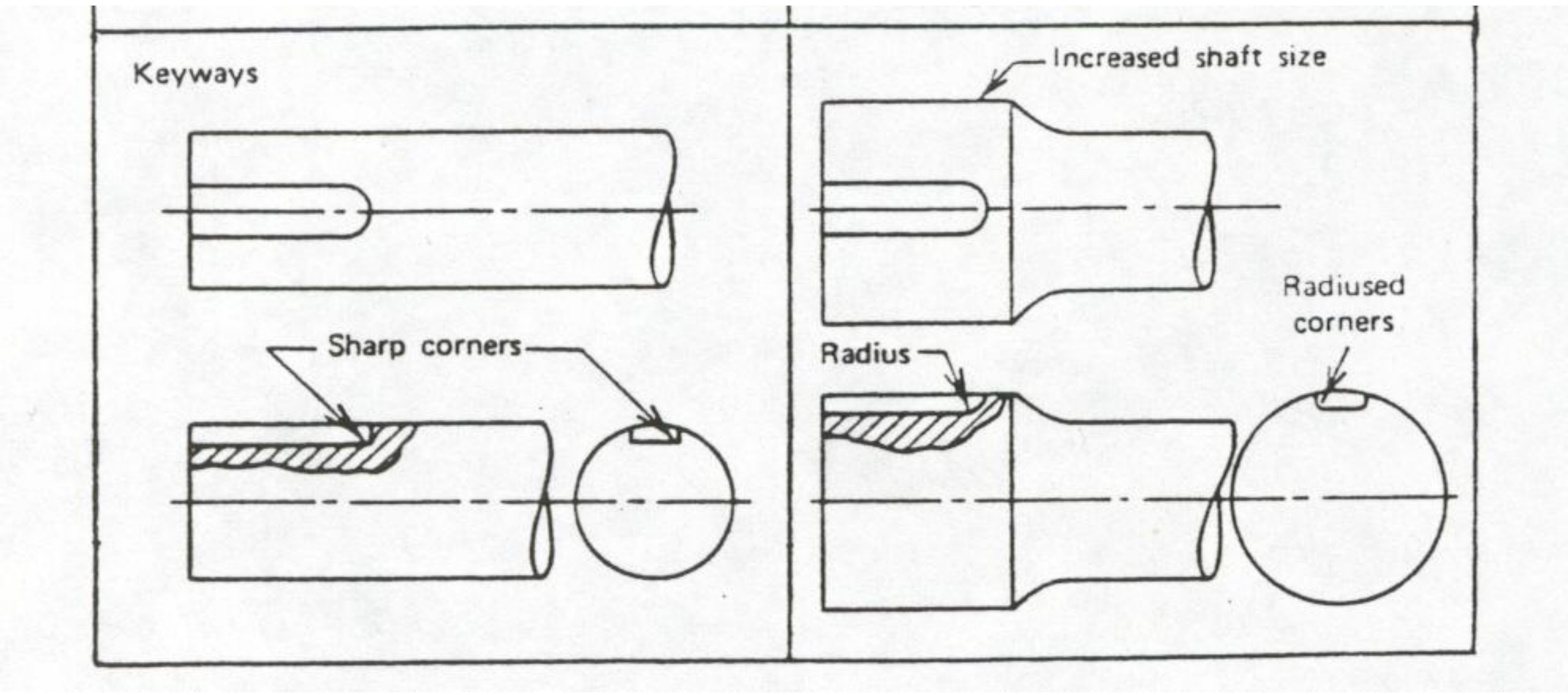



Poor Fatigue Strength
Improved Fatigue Strength
Shoulders
Sharp corners
Large fillet radius
Undercut fillet
with fitted
collar
Undercut
radiused
fillets
Stress–
relieving
grooves


Holes
Enlarged section at hole
Stress–relieving grooves
Grooves
Stress–relieving grooves


Poor Fatigue Strength
Improved Fatigue Strength
Splines
Increased shaft size
Sharp corners
Radiused fillet


Fitted Assemblies
Shaft
Wheel, gear, etc.
Generous
radius
Increase in
journal size
Grooves in
shaft
Fillets on
hub
Grooves in
hub


Keyways
Sharp corners
Increased shaft size
Radius
Radiused corners

S-N APPROACH

FOR

NOTCHED MEMBERS

S-N APPROACH FOR NOTCHED MEMBERS

- Notch Sensitivity and the Fatigue Notch Factor, K_f
- Effects of Stress Level on Notch Factor
- Mean Stress Effects and Haigh Diagrams
- Example of Life Estimation with *S-N* Approach

S-N APPROACH FOR NOTCHED MEMBERS
(Notch Sensitivity and Fatigue Notch Factor, K_f)

- The effect of the notch in the stress-life approach is taken into account by modifying the unnotched *S-N* curve through the use of the fatigue notch factor, K_f.

- Notched fatigue strength not only depends on the stress concentration factor, but also on other factors such as the notch radius, material strength, and mean and alternating stress levels.

- The ratio of smooth to net notched fatigue strengths, based on the ratio of alternating stresses is called K_f.

$$K_f = \text{(Smooth fatigue strength) / (Notched fatigue strength)}$$

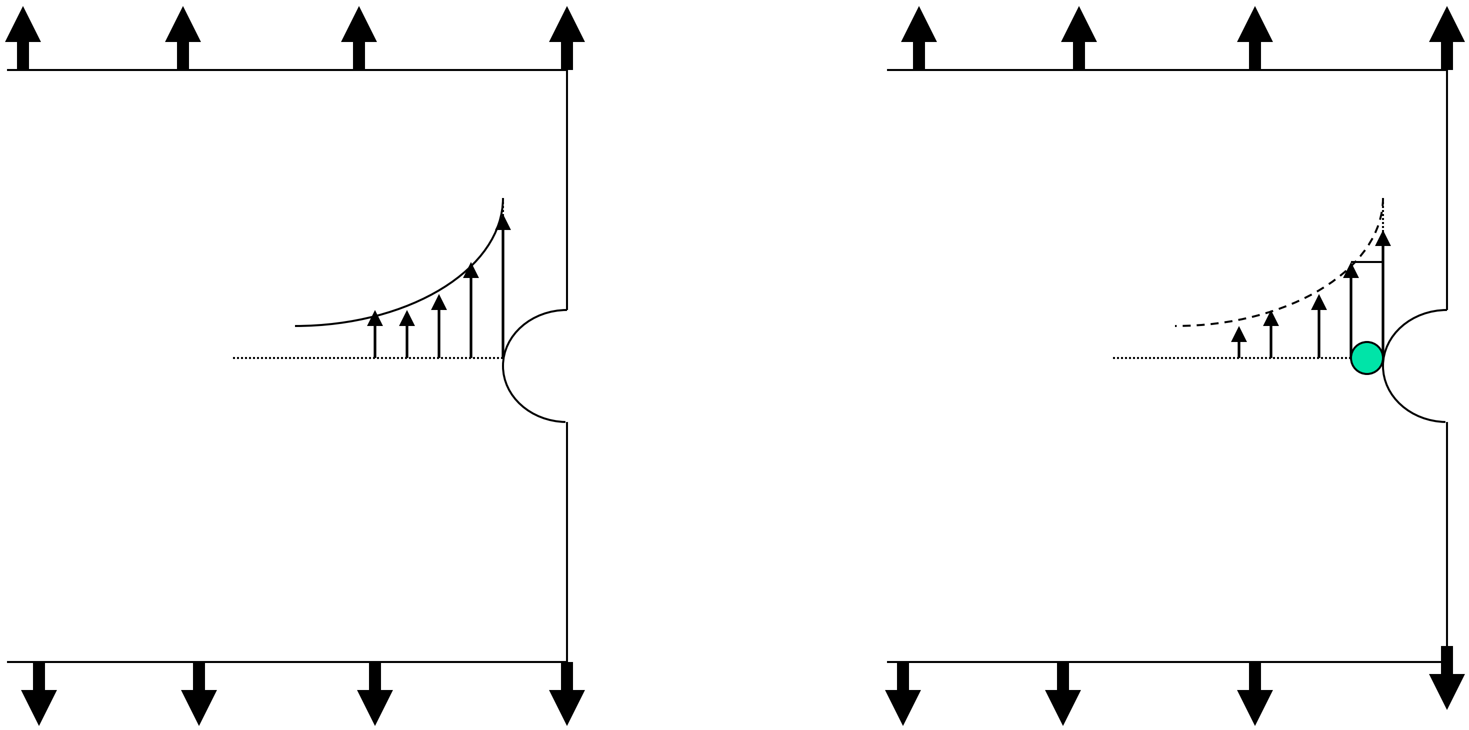
S-N APPROACH FOR NOTCHED MEMBERS (**Notch Sensitivity and Fatigue Notch Factor, K_f**)

- The fatigue notch factor, K_f, is not necessarily equal to the elastic stress concentration factor.

- As a base for estimating the effect of other parameters we estimate the fatigue notch factor K_f for zero mean stress and long life (10^6-10^8 cycles).

S-N APPROACH FOR NOTCHED MEMBERS
(Notch Sensitivity and Fatigue Notch Factor, K_f)

- The difference between K_f and K_t is related to:
  - Stress gradient:
    - The notch stress controlling the fatigue life is not the maximum stress on the surface of the notch root, but an average stress acting over a finite volume of the material at the notch root. This average stress is lower than the maximum surface stress, calculated from K_t
    - When small cracks nucleate at the notch root, they grow into regions of lower stress due to the stress gradient.
  - Localized plastic deformation at the notch root:
    - The localized plastic deformation and notch blunting effect due to yielding at the notch root reduces the notch root stress, particularly at short lives.



S-N APPROACH FOR NOTCHED MEMBERS
(Notch Sensitivity and Fatigue Notch Factor, K_f)

- Values of K_f for R = -1 generally range between 1 and K_t, depending on the **notch sensitivity of the material, *q***, which is defined by:

$$q = \frac{K_f - 1}{K_t - 1}$$

- A value of $q = 0$ (or $K_f = 1$) indicates no notch sensitivity, whereas a value of $q = 1$ (or $K_f = K_t$) indicates full notch sensitivity.

- The fatigue notch factor can then be described in terms of the material notch sensitivity as

$$K_f = 1 + q\,(K_t - 1)$$

S-N APPROACH FOR NOTCHED MEMBERS
(Notch Sensitivity and Fatigue Notch Factor, K_f)

- **Neuber** has developed the following approximate formula for the **notch factor** for *R* = -1 loading:

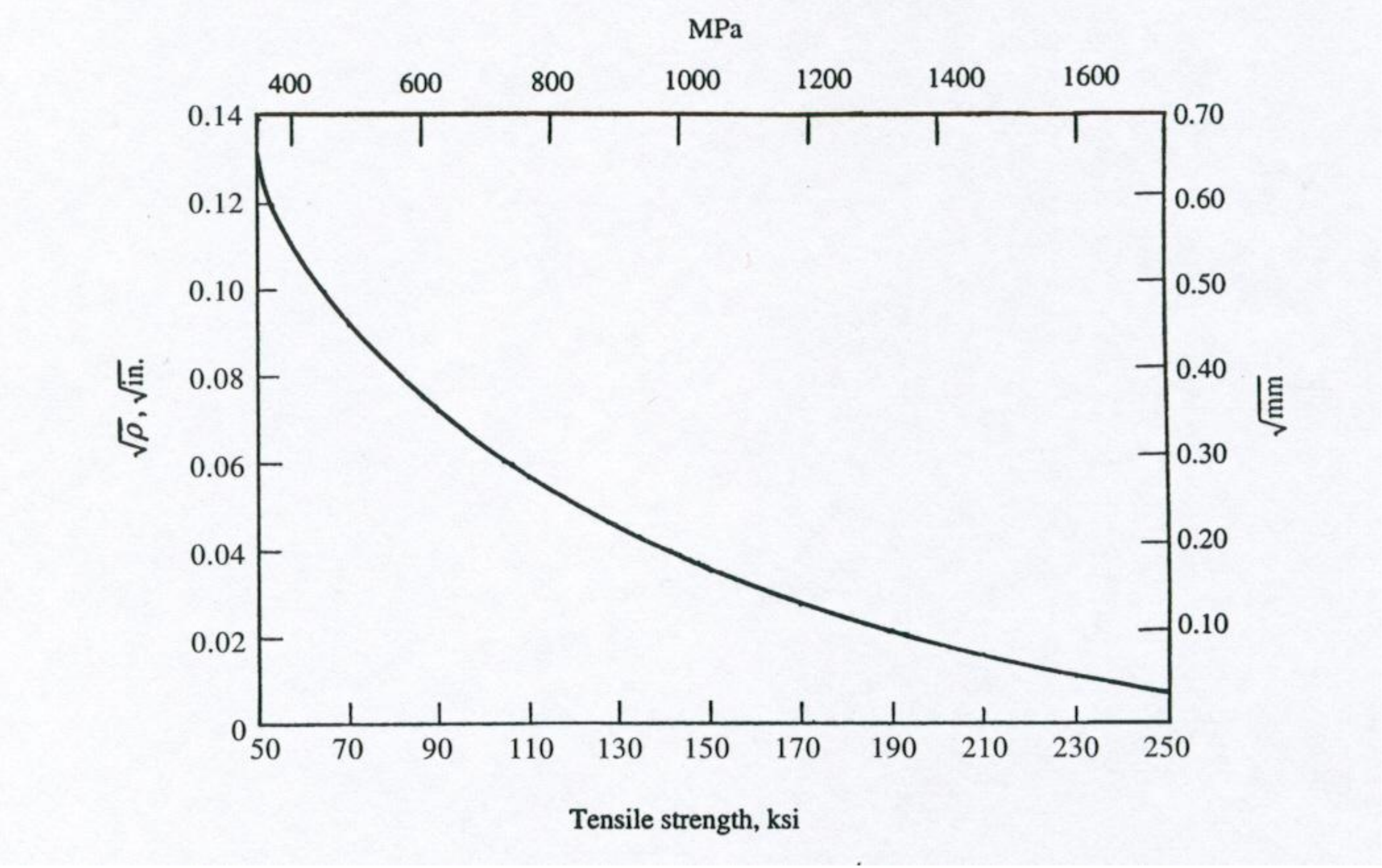
$$q = \frac{1}{1 + \sqrt{\frac{\rho}{r}}} \quad \text{or} \quad K_f = 1 + \frac{K_t - 1}{1 + \sqrt{\frac{\rho}{r}}}$$

  where *r* is the radius at the notch root.

- The characteristic length ρ depends on the material. Values of $\sqrt{\rho}$ for steel alloys are shown in Fig. 7.7, and a few values of ρ **for aluminum** alloys are given as follows:

S_u, MPa (ksi)	150 (22)	300 (43)	600 (87)
ρ, mm (in.)	2 (0.08)	0.6 (0.025)	0.4 (0.015)

Values of $\sqrt{\rho}$ for steel alloys



S-N APPROACH FOR NOTCHED MEMBERS (Notch Sensitivity and Fatigue Notch Factor, K_f)

- **Peterson** has observed that good approximations for R = -1 loading can also be obtained by using the somewhat similar formula:

$$q = \frac{1}{1 + \frac{a}{r}} \quad \text{or} \quad K_f = 1 + \frac{K_t - 1}{1 + \frac{a}{r}}$$

where a is another material characteristic length.

- An empirical relationship between S_u and a for steels is given as:

$$a = 0.0254\left(\frac{2070}{S_u}\right)^{1.8}$$ with S_u in Mpa and a in mm

or

$$a = 0.001\left(\frac{300}{S_u}\right)^{1.8}$$ with S_u in ksi and a in inches

- For aluminum alloys, a is estimated as 0.635 mm (0.025 in.).

S-N APPROACH FOR NOTCHED MEMBERS
(Notch Sensitivity and Fatigue Notch Factor, K_f)

- The formulas to estimate K_f, such as those by Neuber and Peterson, are empirical in nature.

- These formulas express the fact that for large notches with large radii we must expect K_f to be almost equal to K_t, but for small sharp notches we may find $K_f << K_t$ (little notch effect) for metals with ductile behavior, although K_f remains large for high strength metals.

- In general, hard metals are usually more notch sensitive than softer metals.

S-N APPROACH FOR NOTCHED MEMBERS (**Effects of Stress Level** on Notch Factor)

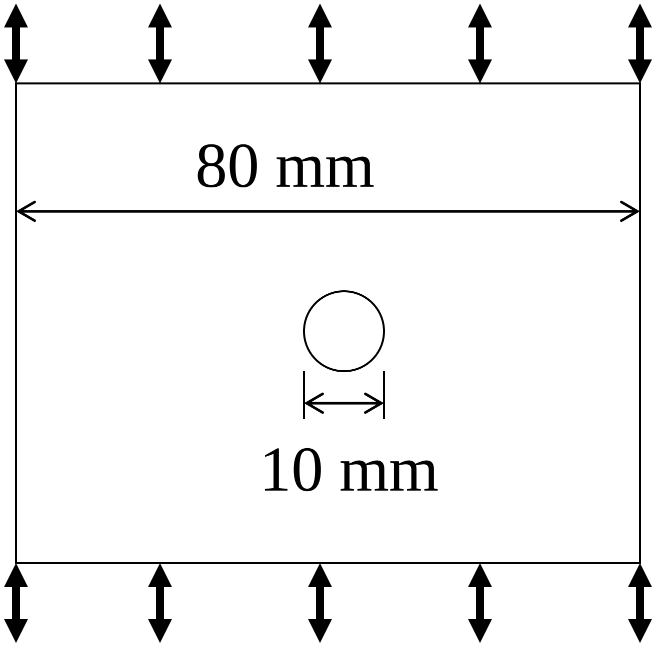
- In the absence of data the behavior of notched parts must be estimated.
  - For fatigue life of 10^6 to 10^8 cycles with R = -1 we can estimate the notched fatigue strength as S_f/K_f.
  - At 1 cycle (approximately a tensile test) the monotonic tensile strength of the notched part for a metal behaving in a ductile manner can be estimated to be equal to the strength of the smooth part in monotonic testing.
  - A straight line between these points, Basquin's equation, in a log *S*-log *N* plot is a reasonable approximation unless other data are available.

S-N APPROACH FOR NOTCHED MEMBERS (**Effects of Stress Level** on Notch Factor)

- An ***S-N* curve for fully reversed stresses for notched parts** can thus be estimated from the following data:
  - The ultimate tensile strength, S_u, or true fracture strength, σ_f, of the material.
  - The long life fully reversed fatigue strength, S_f, for smooth specimens of comparable size.
  - The material characteristic length a or ρ.
  - The elastic stress concentration factor, K_t, and radius, r, of the notch.

EXAMPLE

- For an 80 mm wide sheet of 1020 hot-rolled steel with a 10 mm central hole, construct the *S-N* curve.



EXAMPLE

From Table A.1:

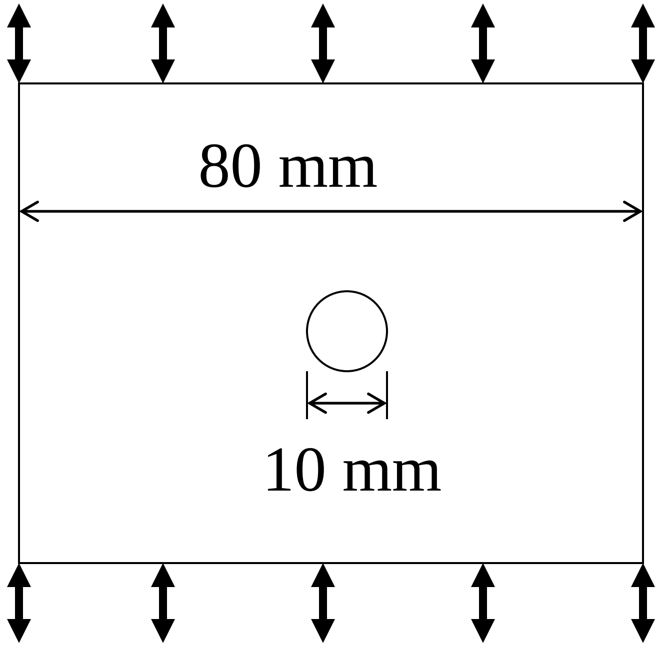
S_f = 241 MPa at approximately 10^6 cycles

S_u = 448 MPa.

Define the smooth line from 448 MPa at 1 cycle to 241 MPa at 10^6 cycles.

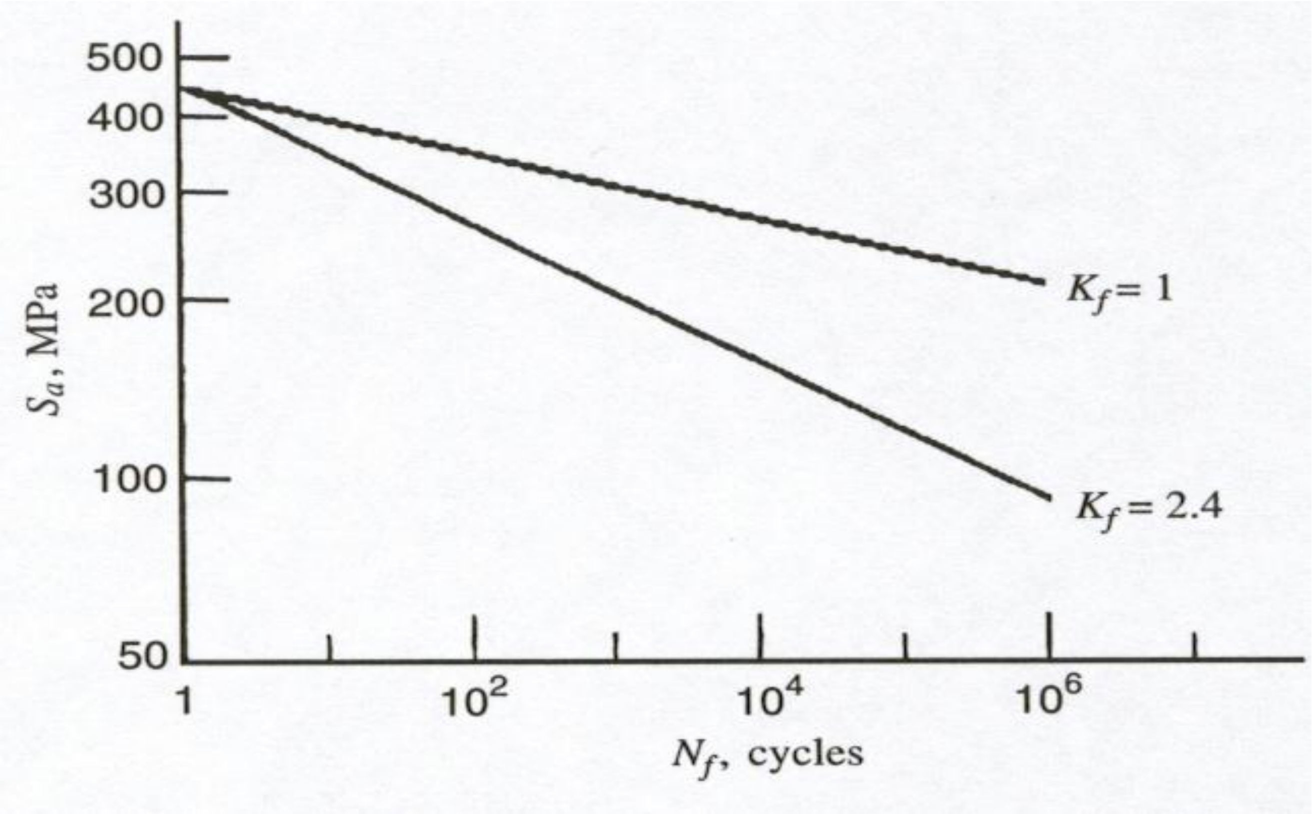
From Fig. 7.1: K_t = 2.7, and From Fig. 7.7, ρ = 0.24 mm. Then:

$$K_f = 1 + \frac{K_t - 1}{1 + \sqrt{\frac{\rho}{r}}} = 1 + \frac{2.7 - 1}{1 + \sqrt{\frac{0.24}{5}}} = 2.4$$



EXAMPLE

- The *S-N* line for the sheet with the hole then goes from 448 MPa at 1 cycle to 241/2.4 = 100 MPa at 10^6 cycles.



S-N APPROACH FOR NOTCHED MEMBERS (Effects of Stress Level on Notch Factor)

- It should be noted that for metals behaving in a brittle manner, the notch effect at short lives is usually more pronounced than that presented in Fig. 7.8.

- An alternative estimate of the *S-N* curve for a notched member made of a ductile material assumes equal fatigue strengths of the notched and smooth members at 10^3 reversals (discussed later in the next section).

S-N APPROACH FOR NOTCHED MEMBERS

Mean Stress Effects
and
Haigh Diagram

S-N APPROACH FOR NOTCHED MEMBERS (**Mean Stress Effects** and Haigh Diagram)

- Figure 7.9 shows the fatigue strengths of smooth and notched specimens of 7075-T6 AL alloy at 10^4 and at 10^7 cycles plotted versus the mean stress.

- Elastic stress concentration factor was 3.4.

- From Fig. 7.9, values of the fatigue notch factor, K_f, are shown below.

At 10^4 cycles	At 10^7 cycles
At zero mean stress	
$K_f = 51/22 = 2.3$	$K_f = 22/10 = 2.2$
At 172 MPa mean stress	
$K_f = 42/13 = 3.2$	$K_f = 17/3 = 5.7$

- Obviously not only is the fatigue notch factor not equal to the elastic stress concentration factor, but it also changes with the mean stress and cycles to failure.

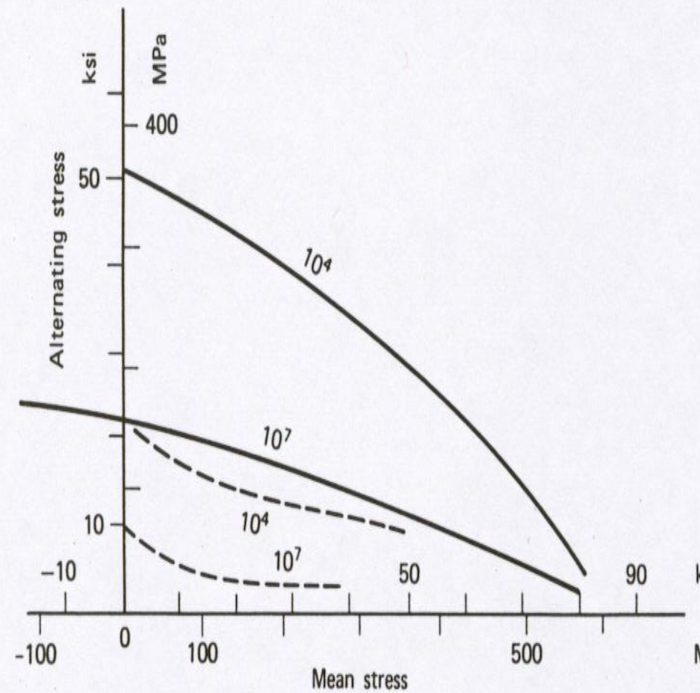


Figure 7.9 Constant life diagram for 7075-T6 wrought aluminum alloy with S_u = 570 MPa (82 ksi) [2]. (—) Unnotched, (---) notched, K_t = 3.4.

S-N APPROACH FOR NOTCHED MEMBERS (Mean Stress Effects and **Haigh Diagram**)

- Figure 7.10 shows lines for median fatigue life of 10^6 cycles for smooth parts and for notched parts with K_f = 2.9 for this material in terms of alternating stress S_a versus mean stress S_m.

- Note the great variation in the ratio K_f of notched parts as compared to the smooth parts.

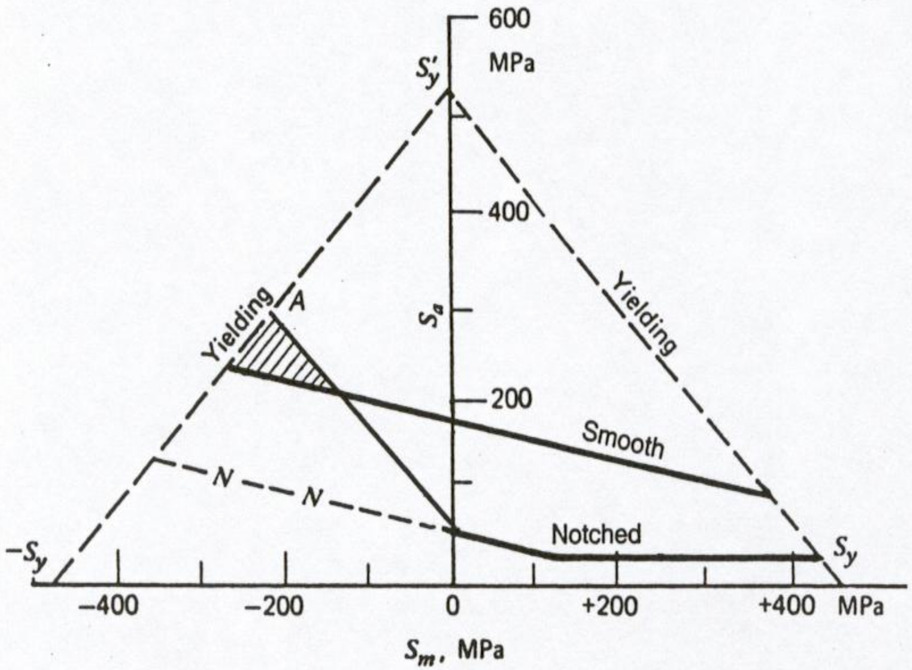


Figure 7.10 Haigh diagram for 7075-T6 aluminum alloy at 1 million cycles, with and without a notch.

S-N APPROACH FOR NOTCHED MEMBERS (Mean Stress Effects and **Haigh Diagram**)

- On the compression side K_f decreases to less than 1 at greater compressive mean stress, which means that a part with a groove may be stronger than a smooth part! (due to compressive residual stresses discussed in Ch 8).

- The diagram in Fig. 7.10 is typical. Similar diagrams can easily be constructed for other materials and other values of K_f.

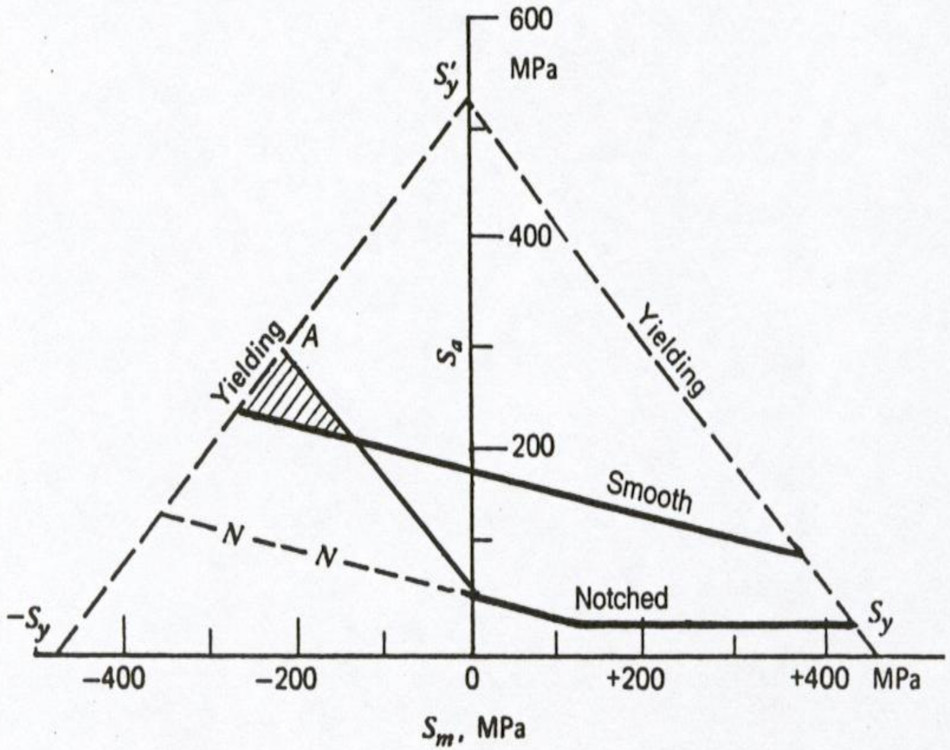


Figure 7.10 Haigh diagram for 7075-T6 aluminum alloy at 1 million cycles, with and without a notch.

S-N APPROACH FOR NOTCHED MEMBERS (Mean Stress Effects and **Haigh Diagram**)

- The important points to remember are:
  - Mean stress has more effect in notched parts than in smooth specimens.
  - Tensile mean stress can increase the fatigue notch factor K_f above the stress concentration factor K_t and can be fatal in fatigue loading.
  - Compressive mean stress can significantly reduce and even eliminate the effects of stress concentrations and save parts.
  - Mean stresses inherent in the unloaded part due to residual stresses are often much greater than mean stresses caused by external loads.

S-N APPROACH FOR NOTCHED MEMBERS (Mean Stress Effects and **Haigh Diagram**)

- The *S-N* approach for the combined effects of the mean stress and the notch is based on the use of available or estimated **Haigh diagrams** (constant life diagrams), such as that shown in Fig. 7.10.
  - From such diagrams one point of an *S-N* curve is obtained.
  - A second point for the *S-N* curve is obtained from knowledge or estimate of the stress corresponding to a very short life, usually either 1 or 1000 cycles.
  - These points are joined by a straight line on log *S*-log *N* coordinates.

S-N APPROACH FOR NOTCHED MEMBERS (Mean Stress Effects and **Haigh Diagram**)

- To estimate a Haigh diagram, the following data must be either available or estimated:
  - The monotonic yield strength, S_y.
  - The cyclic yield strength, S_y'.
  - The unnotched fully reversed fatigue limit, S_f, or the fatigue strength at about 10^6 to 10^8 cycles.
  - The true fracture strength, σ_f.
  - The fully reversed long life fatigue notch factor, K_f.
  - The critical alternating tensile stress, S_{cat}.
    - The critical alternating tensile stress, S_{cat}, is the stress below which cracks will not propagate.
    - It is estimated as 70 MPa (10 ksi) for hard steel, 30 MPa (4 ksi) for mild steel, and 20 MPa (3 ksi) for high strength aluminum. However, if any margin for safety is used, S_{cat} can be taken as zero in design.

S-N APPROACH FOR NOTCHED MEMBERS (Mean Stress Effects and **Haigh Diagram**)

- **Construction of the Haigh diagram** is shown in Fig. 7.11.
  - Any combination of mean and alternating stresses outside the triangle from $-S_y$ to S_y' to $+S_y$ corresponds to gross yielding.
  - Any combination above the line *AB* will produce a median fatigue life less than 10^6 to 10^8 cycles for smooth parts.
  - For a part with a notch, the estimated Haigh diagram is shown by lines *FCDE*.
  - The presence of tensile mean stress reduces the amount of alternating stress that can be tolerated.
  - Maximum alternating stress can be tolerated with a sufficient compressive mean stress (Point *F*).

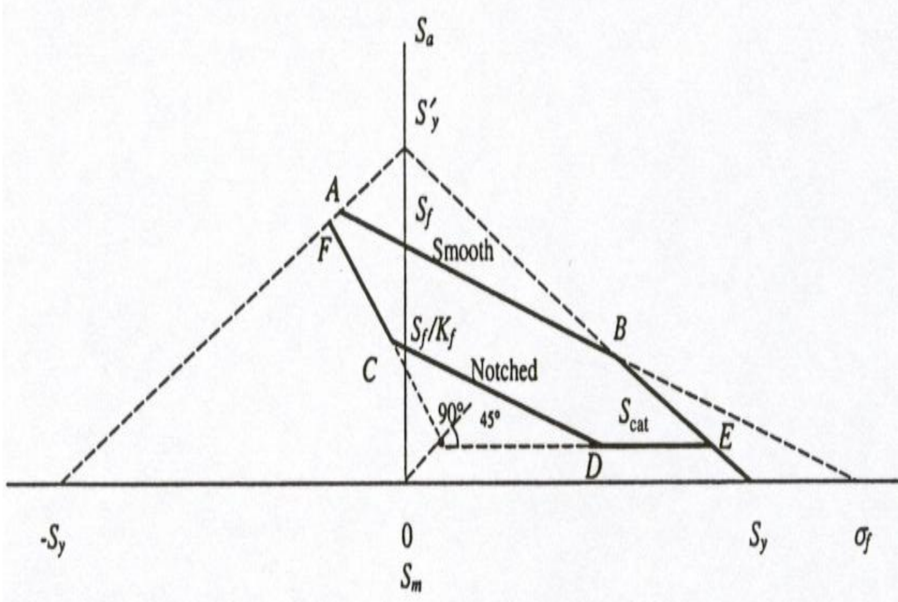


Figure 7.11 Haigh diagram.

S-N APPROACH FOR NOTCHED MEMBERS (**Mean Stress Effects** and Haigh Diagram)

- A simple approach for estimating the long life fatigue strength with a mean stress which does not require construction of the Haigh diagram is use of the **modified Goodman equation**.
  - For a notched member, the long life smooth fatigue strength is simply divided by the fatigue notch factor, K_f:

$$\frac{S_a}{\left(S_f / K_f\right)} + \frac{S_m}{S_u} = 1$$

- The estimates for both smooth and notched parts based on the modified Goodman equation along with the yield limits are shown in Fig. 7.12.

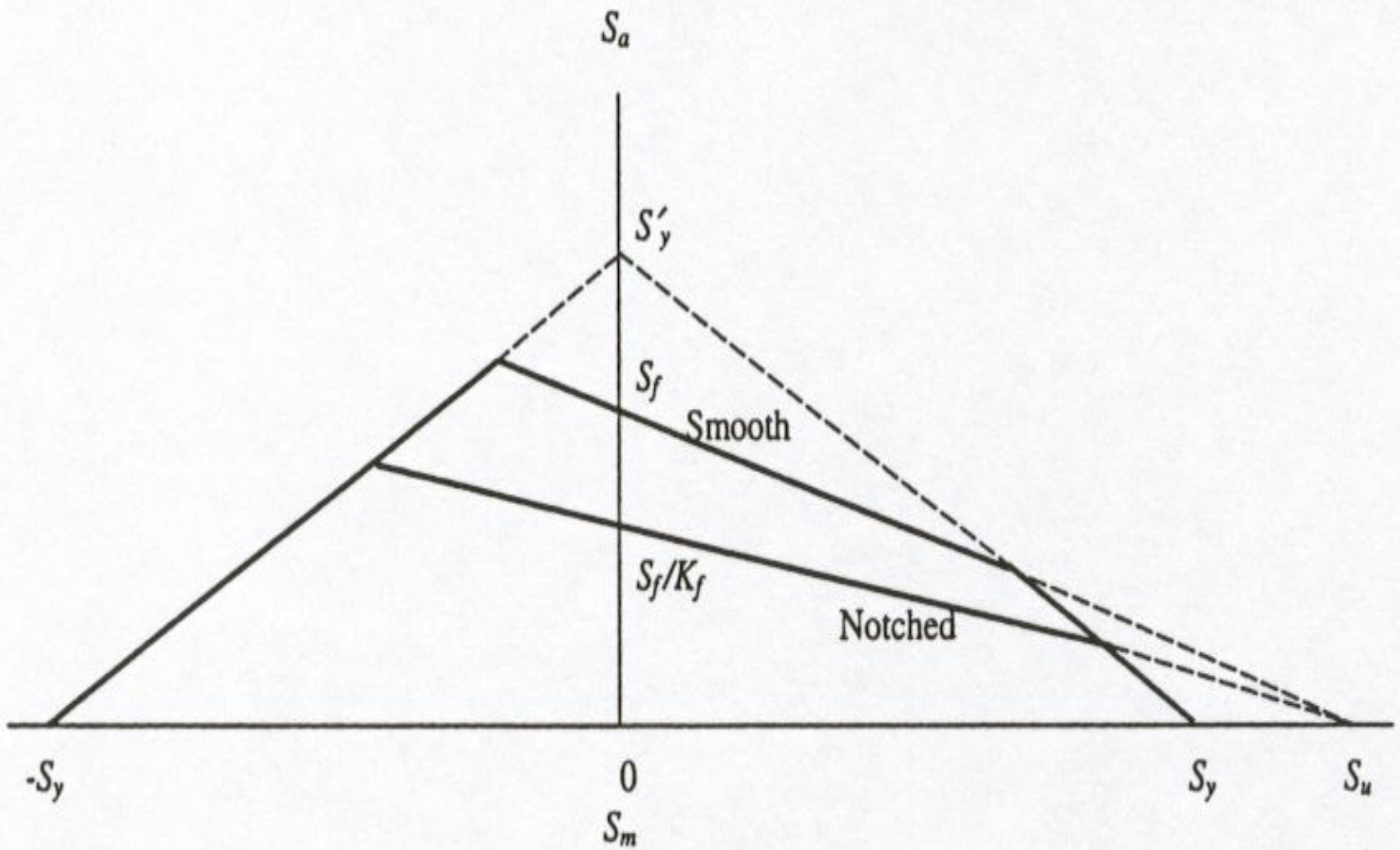


Figure 7.12 Modified Goodman diagram for smooth and notched members.

S-N APPROACH FOR NOTCHED MEMBERS (**Mean Stress Effects** and Haigh Diagram)

- With diagrams like those in Figs. 7.11 or 7.12 the **long-life** fatigue strength of parts with notches for any combination of mean and alternating stresses can be estimated.

- For an estimate of a **short life** at a high stress:
  - Static fracture is one point that can be used, based on a monotonic fracture test. A conservative estimate for metals with ductile behavior equates the nominal fracture stress in the part to S_u, or σ_f.
  - Another prediction can be obtained by assuming that smooth and notched parts of metals with ductile behavior have equal nominal fatigue strengths at 1000 reversals or 500 cycles.

- One can **interpolate** by assuming a straight line between these points on an *S-N* diagram on logarithmic scales (Basquin's Eq.).

EXAMPLE OF S-N APPROACH FOR A NOTCHED MEMBER WITH MEAN STRESS

- Construct S-N lines for
  - (a) completely reversed loading
  - (b) loading with minimum nominal stress of 50 MPa
- Material is QT hot-rolled ROC-100 steel Table A.2:
  - S_u = 931 MPa, S_y = 883 MPa,
  - σ_f = 1330 MPa, S_y' = 600 MPa,
  - σ_f' = 1240 MPa, and b = -0.07.
- $K_t = 3$
- The nominal stress $S = P/A + Mc/I$ = 11.25P (MPa), where P is the load in kN.

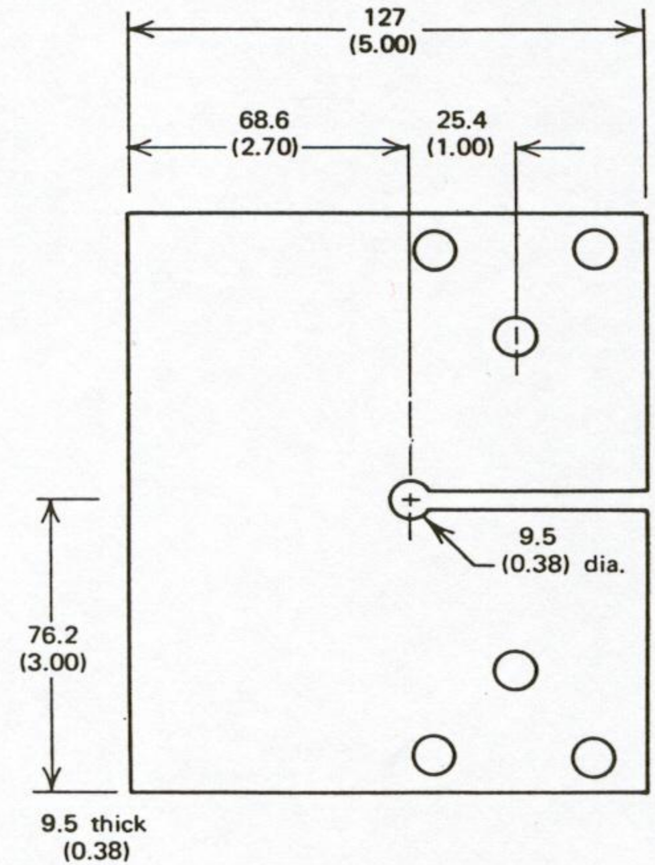


Figure 7.13 Test specimen design, mm (in.).

EXAMPLE OF S-N APPROACH FOR A NOTCHED MEMBER WITHOUT MEAN STRESS

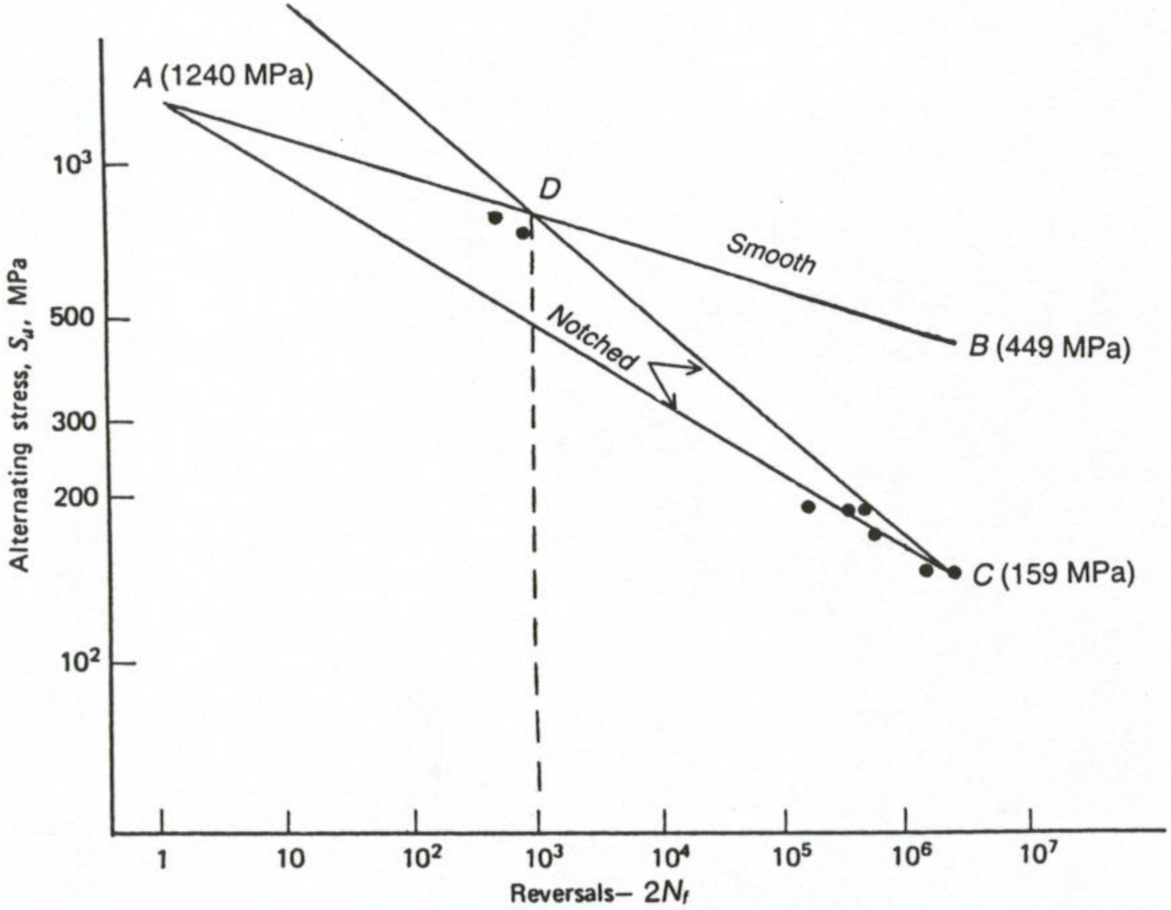


Figure 7.14 Estimated S–N curves and measured fatigue life for fully reversed ($R = -1$) loading.

EXAMPLE OF S-N APPROACH FOR A NOTCHED MEMBER WITH MEAN STRESS

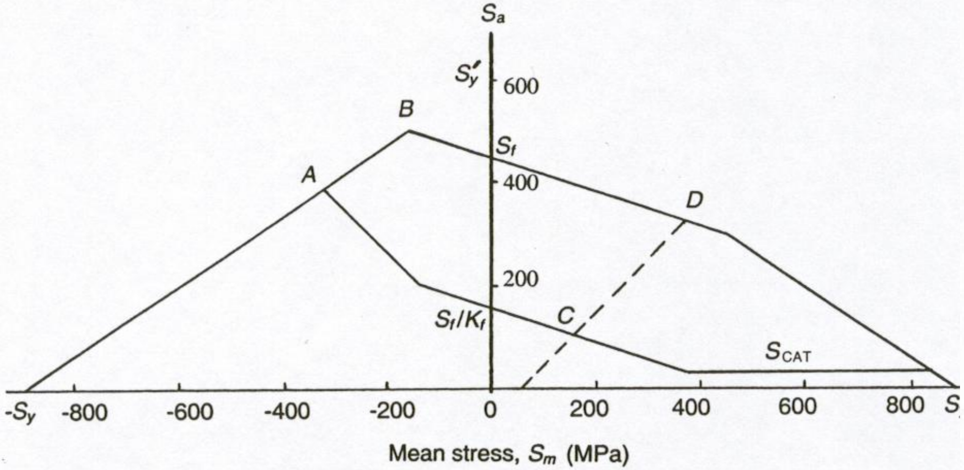


Figure 7.15 Haigh diagram for the example part.

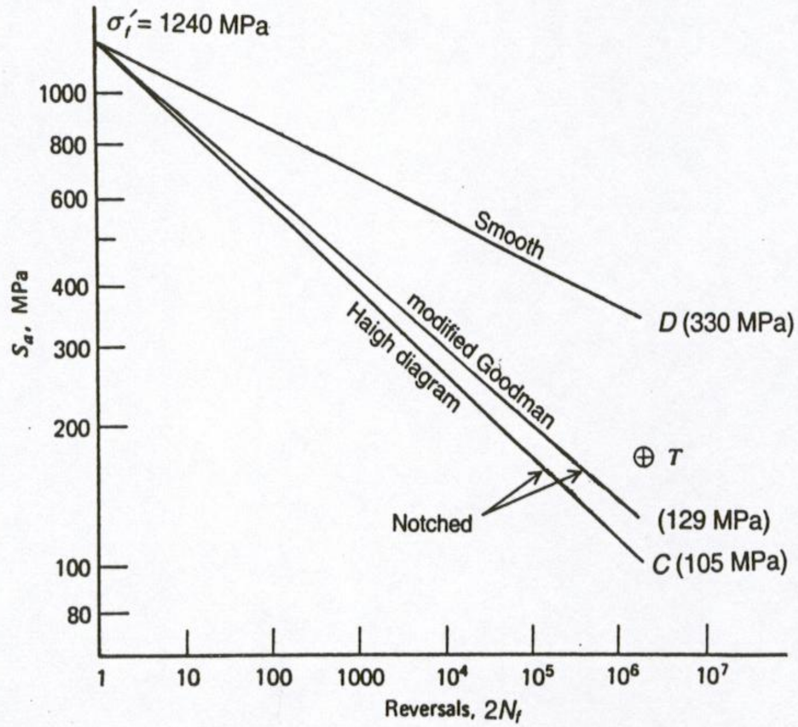


Figure 7.16 S–N lines for smooth and notched parts with tensile mean stress.

STRAIN-LIFE APPROACH

FOR

NOTCHED MEMBERS

STRAIN-LIFE APPROACH

- Notch Stresses and Strains
- Notch Strain Analytical Methods
  - Linear rule
  - Neuber's rule
  - Strain energy density or Glinka's rule
- Plane Stress versus Plane Strain
- Example Problem

STRAIN-LIFE APPROACH

- A common application of the strain-life approach is in fatigue analysis of notched members, because:
  - the deformation of the material at the notch root is often inelastic.
  - notch stresses and strains are explicitly considered in the strain-life approach, whereas the *S-N* approach is only in terms of nominal stresses.

- An example where significant notch plastic deformation and mean stresses are present is in variable amplitude loading with overloads and during low cycle fatigue.

- Application of the strain-life approach involves **two steps**:
  - determination of local (notch) stresses and strains.
  - life prediction using the local stresses and strains, based on the strain-life equation and analysis discussed in Chapter 5.

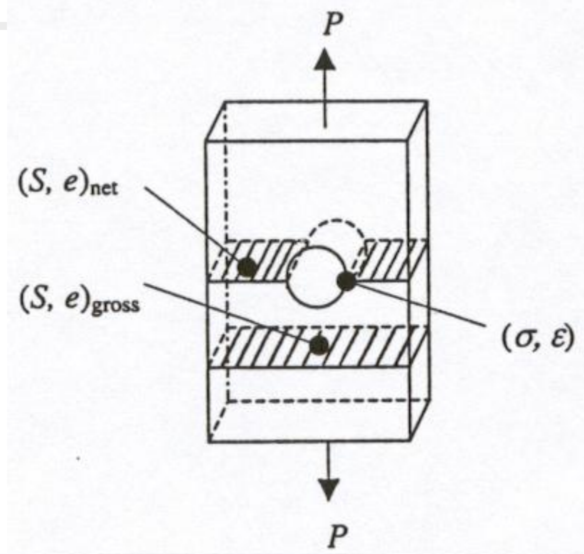
Notch Stresses and Strains

- Notch root and nominal stresses and strains represented by (σ, ε) and (S, e) are defined and shown.

- If the metal at the notch root is strained beyond the yield strength it may strain harden and cyclic harden or soften as discussed in Chs 3 and 5.

- As long as stresses and strains at the notch root remain **elastic** we have:

$$\sigma = K_t\, S \qquad\qquad \varepsilon = K_t\, e$$



S_{net} = net nominal stress = P/A_{net}

e_{net} = net nominal strain

S_{gross} = gross nominal stress = P/A_{gross}

e_{gross} = gross nominal strain

σ = local stress at the notch

ε = local strain at the notch

Notch Stresses and Strains

- The loads on notched parts are often sufficiently high such that the local stress:
  - is considerably above the yield strength
  - is less than $K_t S$
  - stresses are no longer proportional to strains.

- We then define strain and stress concentration factors as:

$$K_{\varepsilon} = \frac{\varepsilon}{e} \qquad K_{\sigma} = \frac{\sigma}{S}$$

- Variations of stress and strain concentration factors with notch stress is shown in Fig. 7.18.

Variations of stress and strain concentration factors with notch stress

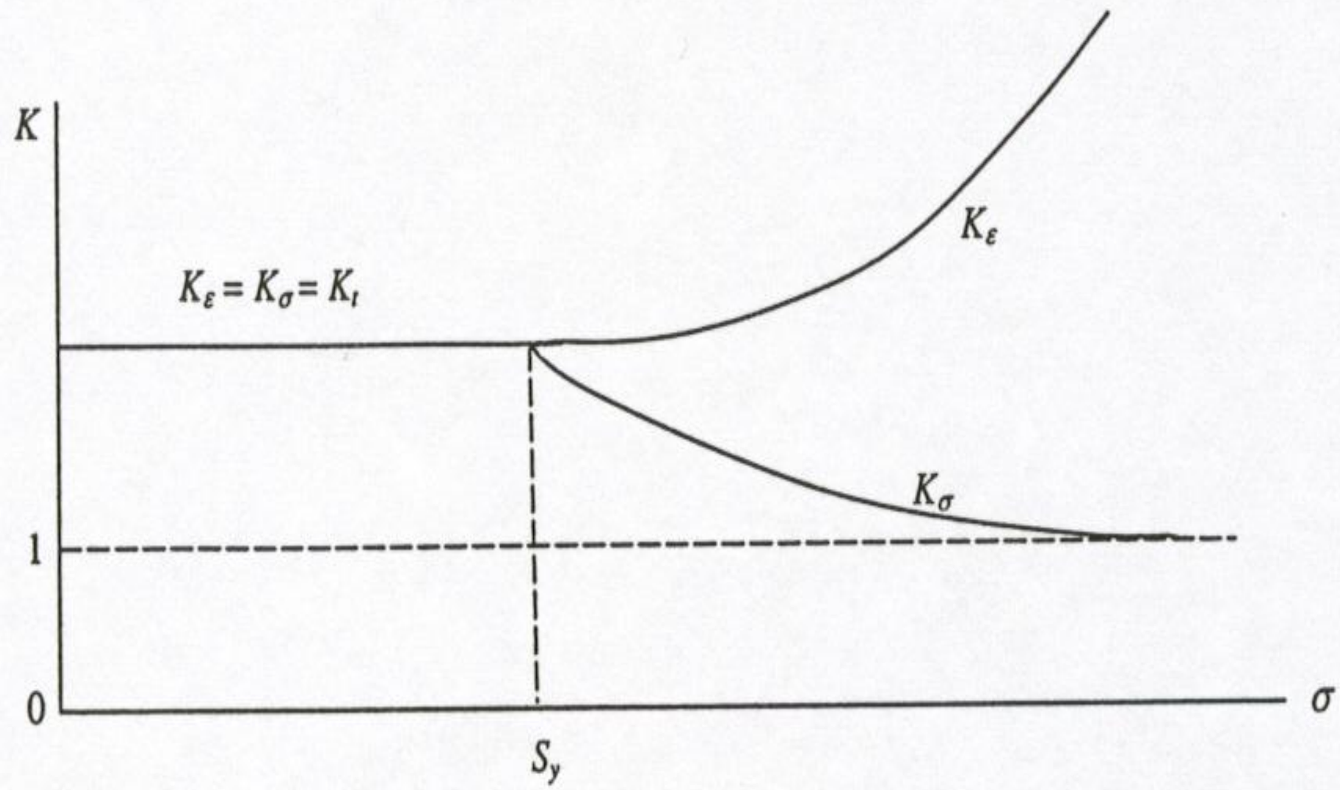


Figure 7.18 Stress and strain concentration factor variations versus notch stress.

Notch Stresses and Strains

- The relation between σ and ε is given by the monotonic stress-strain curve, often represented by the Ramberg-Osgood equation:

$$\varepsilon = \varepsilon_e + \varepsilon_p = \frac{\sigma}{E} + \left(\frac{\sigma}{K}\right)^{1/n}$$

Values for *n* and *K* can be taken from Table A.2.

- Given nominal elastic stress *S* or strain *e*, the local stress σ and the local strain ε at the notch root can be obtained by:
  - experimental methods,
  - finite element methods,
  - analytical models

Notch Stresses and Strains

- Experimental methods were discussed in Section 7.1.

- Finite element method requires
  - small element size in high stress gradient regions
  - a realistic representation of the nonlinear material stress-strain behavior (such as Ramberg-Osgood equation).

- Analytical models
  - Require the value of elastic stress concentration factor, K_t (for complex geometries linear FEA can be used to obtain K_t).
  - Include the **linear rule**, **Neuber's rule**, and **strain energy density or Glinka's rule**.

LINEAR RULE

- The linear rule is expressed as:

$$K_\varepsilon = K_t = \frac{\varepsilon}{e} \qquad \text{or} \qquad \varepsilon = K_t\, e$$

- For nominal elastic behavior, $e = S/E$

- The notch strain, ε, can be computed directly and, if desired, the notch stress, σ, can then be obtained from the stress-strain curve (or Ramberg-Osgood Equation).

- For cyclic loading, notch and nominal stresses and strains are replaced by their respective ranges.

- The linear rule agrees with measurements in plane strain situations, such as circumferential grooves in shafts in tension or bending.

Neuber’s Rule

- Neuber's rule is the most widely used notch stress/strain model.
- It is expressed as: $K_\varepsilon K_\sigma = K_t^2$ or $\varepsilon\,\sigma = K_t^2\, e\, S$
- According to this relation, the geometrical mean of the stress and strain concentration factors under plastic deformation conditions remains constant and equal to the theoretical stress concentration factor, K_t. *See Fig. 7.18*.
- This rule agrees with measurements in plane stress situations, such as thin sheets in tension.
- Application of this rule requires the solution of two simultaneous equations (the above equation which describes a hyperbola, and the stress-strain equation).

Application of Neuber's rule for monotonic loading using a **graphical method**, where point *A* is the solution

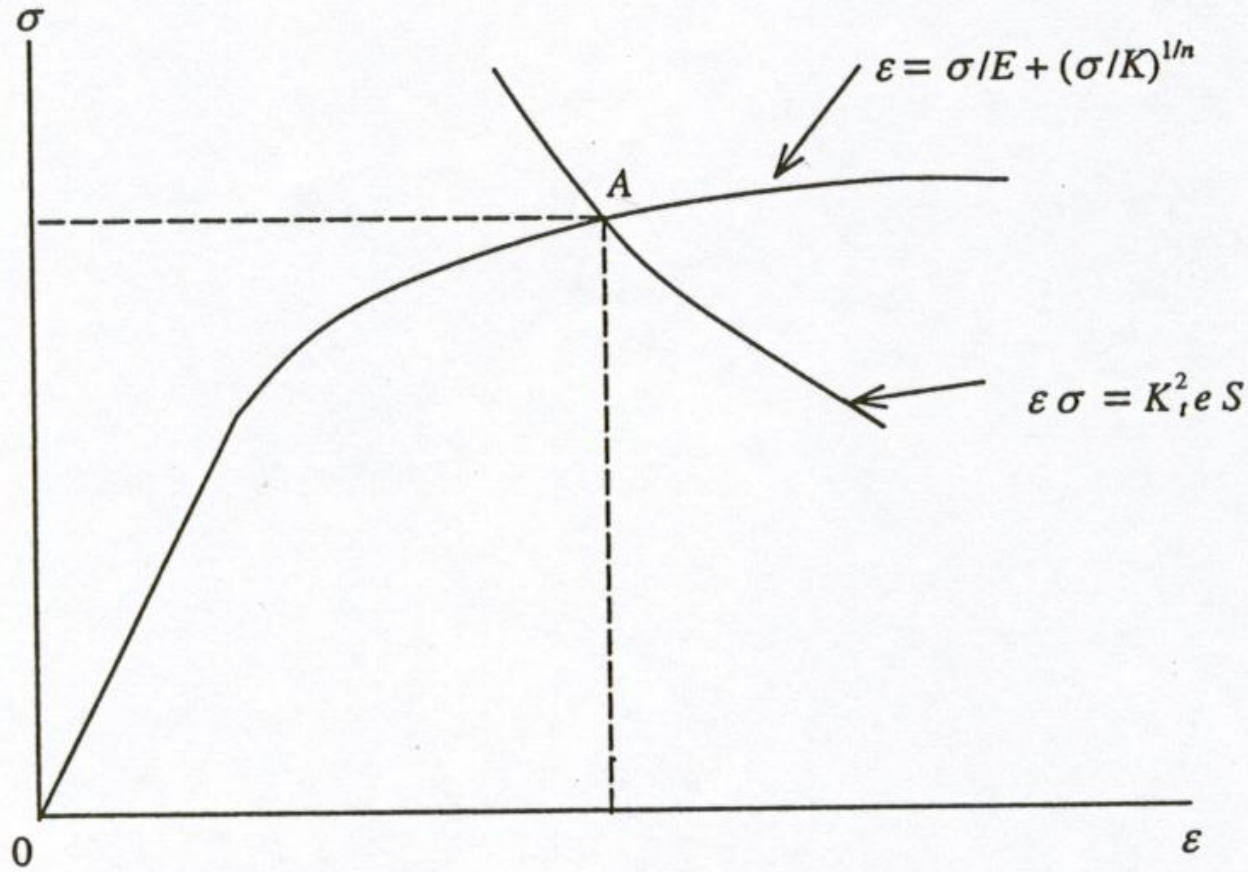


Figure 7.19 Notch strain determination by Neuber's rule.

Neuber's Rule

- For nominal elastic behavior, $e = S/E$:

$\varepsilon\,\sigma = K_t^2\, e\, S$ and Neuber's rule results in:
$$\varepsilon\,\sigma = \frac{(K_t S)^2}{E}$$

- Combining this equation with the stress-strain equation results in

$$\frac{\sigma^2}{E} + \sigma\left(\frac{\sigma}{K}\right)^{1/n} = \frac{(K_t S)^2}{E}$$

This equation can be solved for notch stress, σ, by iteration or numerical techniques

Neuber's Rule

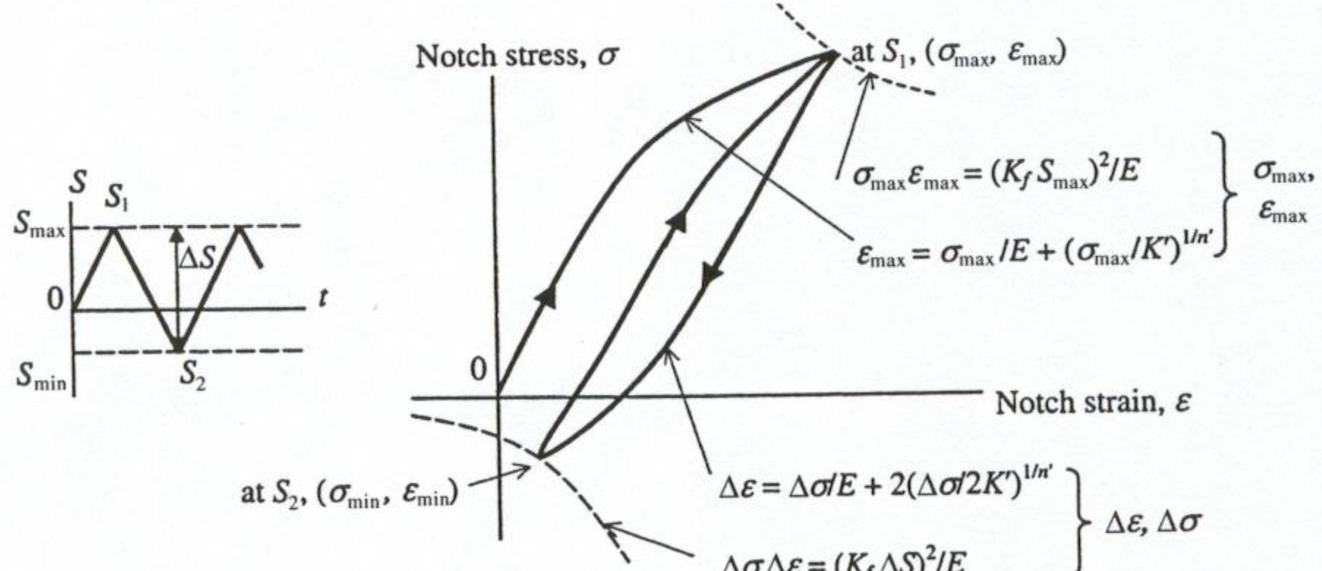
- For **cyclic loading**:
  - The monotonic stress-strain curve is replaced by the hysteresis curve and the strains and stresses are replaced by the strain ranges and stress ranges.
  - Based upon better agreement with experimental fatigue life results, the fatigue notch factor, K_f, is used in place of the theoretical stress concentration factor, K_t.
  - Neuber's rule for computing $\Delta\varepsilon$ then is: $\Delta\varepsilon\,\Delta\sigma = K_f^2\,\Delta e\,\Delta S$
  - For nominal elastic behavior, $\Delta e = \Delta S / E$, and

$$\Delta\varepsilon\,\Delta\sigma = \frac{(K_f\,\Delta S)^2}{E}$$

  - Analogous to monotonic loading: $\dfrac{(\Delta\sigma)^2}{E} + 2\,\Delta\sigma\left(\dfrac{\Delta\sigma}{2K'}\right)^{1/n'} = \dfrac{(K_f\,\Delta S)^2}{E}$

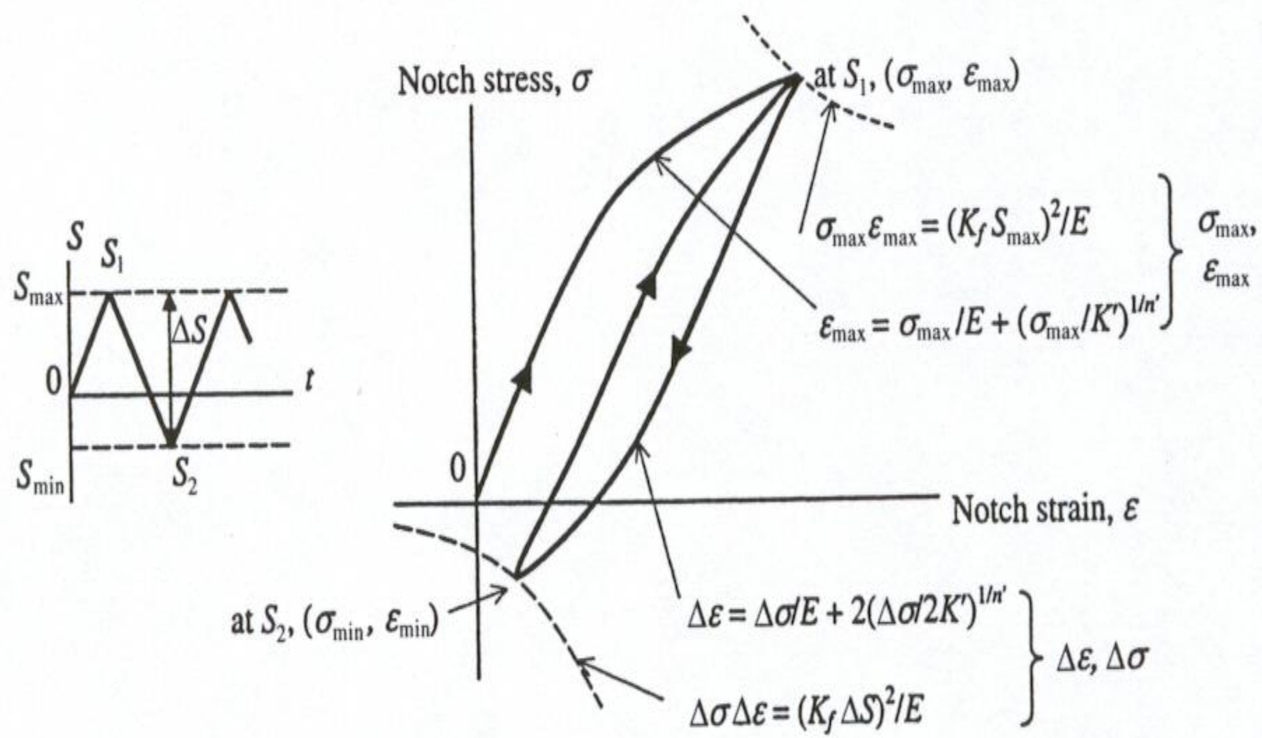
Neuber's Rule

- **Illustration** of Neuber's rule for cyclic loading.
  - Use of the cyclic stress-strain curve assumes stable cyclic behavior.
  - Note that for unloading, the point of reversal S_1 is used as the origin of the Neuber hyperbola and the hysteresis curve, not $\sigma = 0$.
  - For the continued constant amplitude loading, the notch stress and strain will continue to follow the closed hysteresis loop shown.



Neuber's Rule

- Knowing $\Delta\varepsilon$, $\Delta\sigma$, and σ_{max}, we can obtain $\varepsilon_a = \Delta\varepsilon/2$, and $\sigma_m = \sigma_{max} - \Delta\sigma/2$.
- Notch strain amplitude, ε_a, and notch mean stress, σ_m, are then used for life prediction analysis.

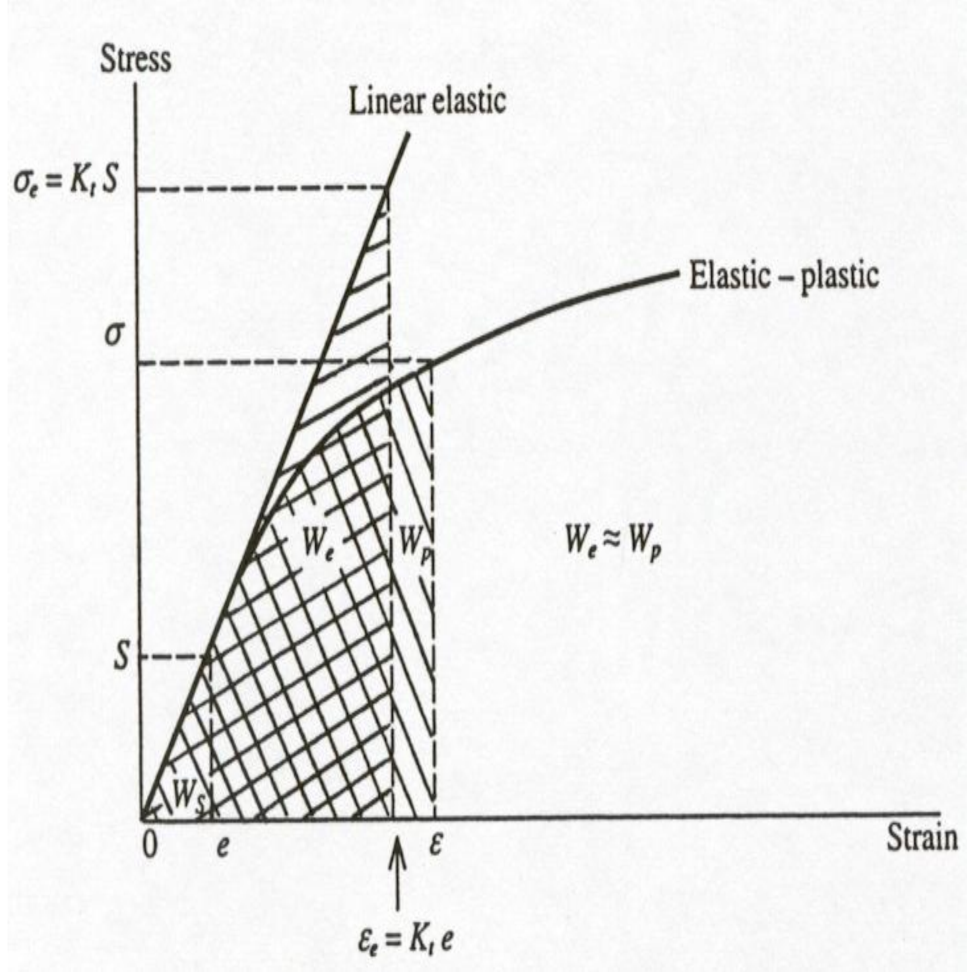


Strain Energy Density or Glinka's Rule

- This rule is based on the assumption that strain energy density at the notch root is nearly the same for linear elastic notch behavior (W_e) and elastic-plastic notch behavior (W_p), as long as the plastic deformation zone at the notch is surrounded by an elastic stress field.

- For nominal elastic stress, S, the nominal strain energy density, W_S, is given by (with $e = S/E$ and $de = dS/E$):

$$W_s = \int_0^e S\, de = \int_0^S \frac{S}{E}\, dS = \frac{S^2}{2E}$$



Strain Energy Density or Glinka's Rule

- At the notch root with a stress concentration factor of K_t, strain energy density assuming **linear elastic** behavior ($\sigma = K_t S$ and $\varepsilon = \sigma / E$) is:

$$W_e = \int_0^{\varepsilon_e} \sigma \, d\varepsilon = \int_0^{\sigma_e} \frac{\sigma}{E} d\sigma = \frac{\sigma_e^2}{2E} = \frac{(K_t S)^2}{2E}$$

- For **elastic-plastic** behavior at the notch root, the stress-strain relationship can be expressed by Ramberg-Osgood equation and the strain energy density is given by:

$$W_p = \int_0^{\varepsilon} \sigma \, d\varepsilon = \frac{\sigma^2}{2E} + \frac{\sigma}{n+1}\left(\frac{\sigma}{K}\right)^{1/n}$$

Strain Energy Density or Glinka's Rule

- Setting $W_p = W_e$ results in:

$$\frac{\sigma^2}{E} + \frac{2\sigma}{n+1}\left(\frac{\sigma}{K}\right)^{1/n} = \frac{(K_t S)^2}{E}$$

  - For a given nominal stress *S*, notch stress σ can be calculated.
  - The only difference with Neuber's rule is the factor [2/(*n*+1)].
  - Since *n* < 1, smaller notch stress (and therefore smaller notch strain) is predicted based on this equation than from Neuber's rule).

Strain Energy Density or Glinka's Rule

- For **cyclic loading**, the equation is written in terms of stress and strain ranges and material monotonic deformation properties (*K* and *n*) are replaced by cyclic deformation properties (*K*' and *n*'):

$$\frac{(\Delta\sigma)^2}{E} + \frac{4\Delta\sigma}{n'+1}\left(\frac{\Delta\sigma}{2K'}\right)^{1/n'} = \frac{(K_t\Delta S)^2}{E}$$

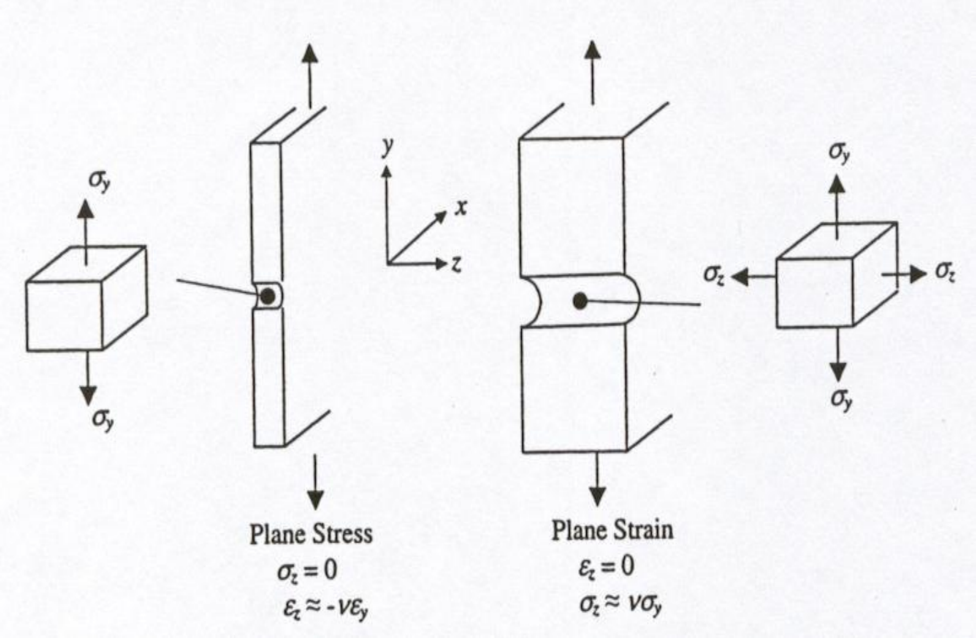
- This equation relates nominal stress range, ΔS, to notch stress range, $\Delta\sigma$.

- Notch strain range, $\Delta\varepsilon$, is then found from the hysteresis loop equation and used in the strain-life equations to find the fatigue life, similar to the procedure for Neuber's rule.

Plane Stress versus Plane Strain

- The notch strain analysis presented assumed a uniaxial state of stress at the notch root. This condition exists for a plane state of stress such as in a thin plate.

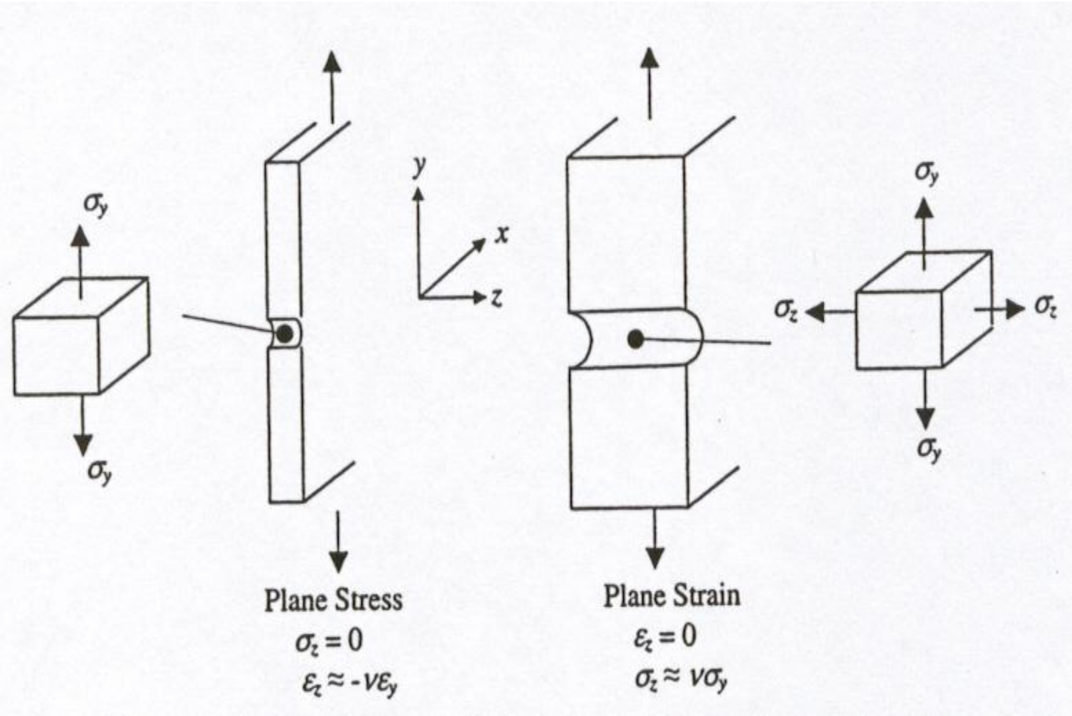
- For plane strain conditions such as in a thick plate, however, the state of stress is no longer uniaxial, because in the thick plate the surrounding elastic material restrains notch deformation in the thickness direction, $\varepsilon_z \approx 0$, resulting in a stress σ_z.



Plane Stress versus Plane Strain

- For the plane strain conditions, a smaller notch strain range and a larger notch stress range in the loading direction results, as compared with the plane stress condition.

- Plane stress and plane strain conditions represent two bounding conditions which occur at the notch root.



EXAMPLE OF LIFE ESTIMATION OF A NOTCHED MEMBER **WITH ε-N APPROACH**

Example of Life Estimation with ε-N Approach

- Notched part shown with $K_t = 3$ and made of RQC-100 steel.

- We want to find:
  - (a) notch stress and strain from a 53.4 kN (12 kip) monotonic load,
  - (b) notch stress and strain after unloading from the monotonic load in part (a) to zero,
  - (c) notch stress and strain amplitudes from constant amplitude alternating loads between 4.45 kN (1 kip) and 44.5 kN (10 kip), and
  - (d) the expected fatigue life to the formation of cracks on the order of 1 mm from the loading in part (c).

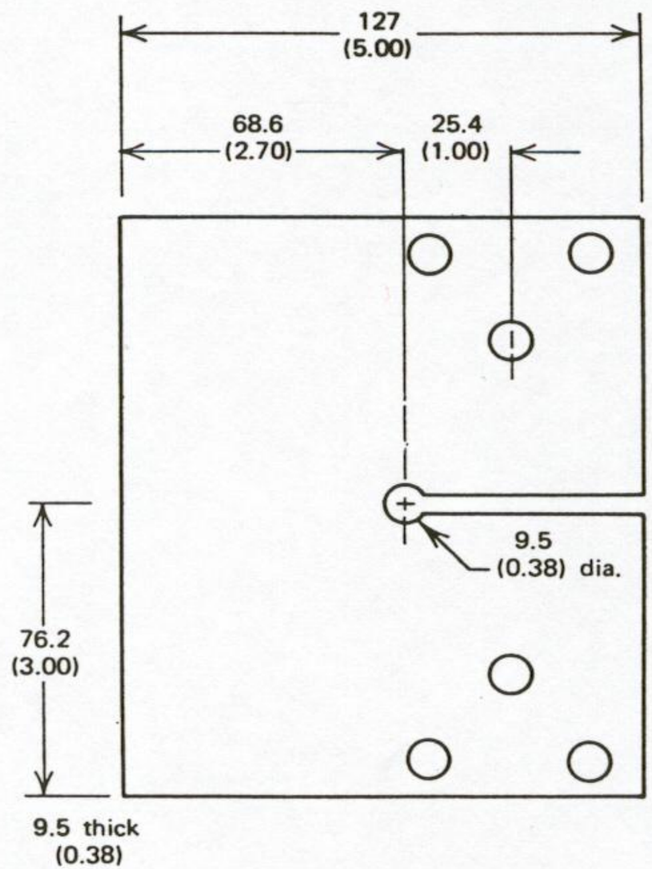


Figure 7.13 Test specimen design, mm (in.).

Example of Life Estimation with ε-N Approach

- The relevant **properties** quoted from Table A.2 are:

$E = 207$ GPa $\quad S_y = 883$ MPa $\quad K = 1172$ MPa $\quad n = 0.06$

$S_y' = 600$ MPa $\quad K' = 1434$ MPa $\quad n' = 0.14$

$\sigma_f' = 1240$ MPa $\quad \varepsilon_f' = 0.66 \quad b = -0.07 \quad c = -0.69$

Example of Life Estimation with ε-N Approach

PART (a): Find notch stress & strain from a 53.4 kN monotonic load

- From Example 7.2.4, $S = 11.25\, P = 11.25\,(53.4) = 600$ MPa
- The nominal monotonic behavior is elastic, since $S_{max} = 600$ MPa is about 2/3 of the yield strength, $S_y = 883$ MPa.
- However, the monotonic notch root behavior is inelastic since $K_t S_{max} = 3 \times 600 = 1800$ MPa > $S_y = 883$ MPa.
- To find the notch root stress and strain using **Neuber's rule**, we have to solve the following two simultaneous equations:

$$\varepsilon\,\sigma = \frac{(K_t S)^2}{E} = \frac{(3 x 600)^2}{207000} = 15.65$$

$$\varepsilon = \frac{\sigma}{E} + \left(\frac{\sigma}{K}\right)^{1/n} = \frac{\sigma}{207000} + \left(\frac{\sigma}{1172}\right)^{1/0.06}$$

resulting in: $\sigma = 903$ MPa and $\varepsilon = 0.0173$

(See Fig. 7.23)

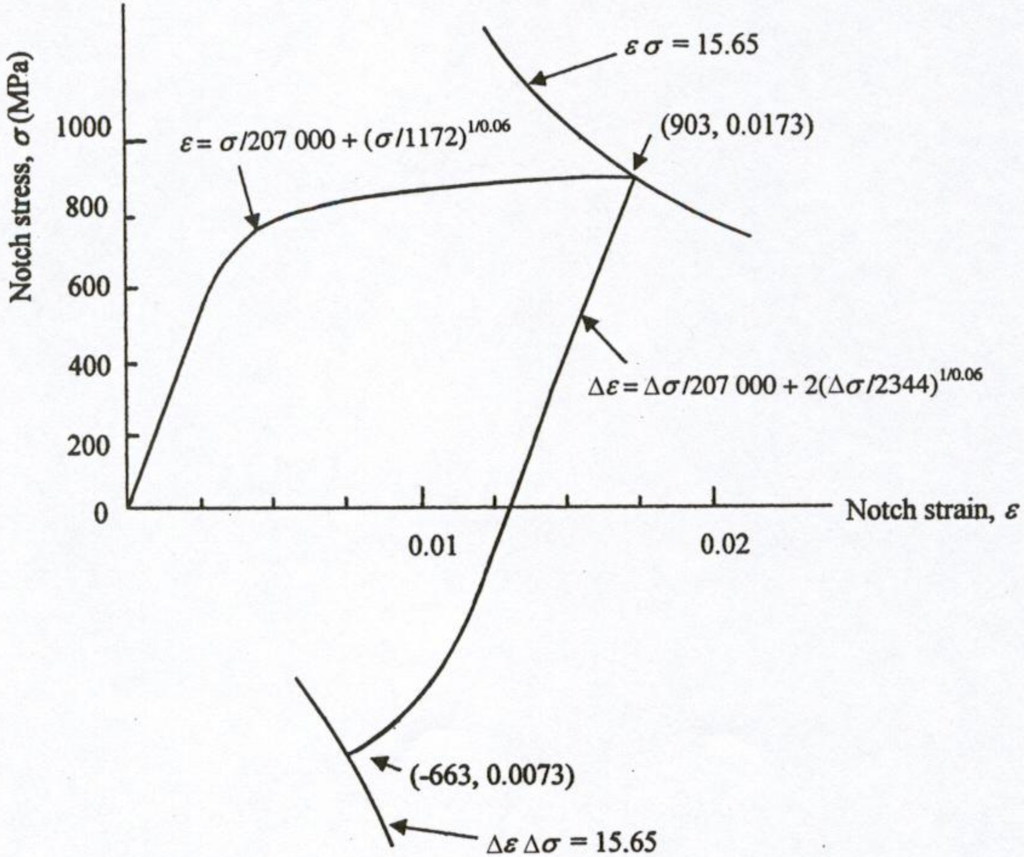


Figure 7.23 Determination of the strain at the root of a notch according to Neuber's rule; material: RQC-100; nominal stress 600 MPa followed by unloading to zero; $K_t = 3$.

Example of Life Estimation with ε-N Approach

- If **strain energy density rule** is used:

$$\frac{\sigma^2}{E} + \frac{2\sigma}{n+1}\left(\frac{\sigma}{K}\right)^{1/n} = \frac{(K_t S)^2}{E}$$

or: $$\frac{\sigma^2}{207000} + \frac{2\sigma}{0.06+1}\left(\frac{\sigma}{1172}\right)^{1/0.06} = \frac{(3x600)^2}{207000} = 15.65$$ resulting in $\underline{\sigma = 872\ \text{MPa}}$

Substituting this value into the stress-strain equation results in:

$$\varepsilon = \frac{\sigma}{E} + \left(\frac{\sigma}{K}\right)^{1/n} = \frac{872}{207000} + \left(\frac{872}{1172}\right)^{1/0.06} = \underline{0.0115}$$

These values are smaller than those predicted by Neuber's rule, as expected.

Example of Life Estimation with ε-N Approach

- If we used the **linear rule**, notch root strain and stress would be calculated as

$$\varepsilon = K_t e = \frac{K_t S}{E} = \frac{3x600}{207000} = \underline{0.0087}$$

$$0.0087 = \frac{\sigma}{207000} + \left(\frac{\sigma}{1172}\right)^{1/0.06}$$ or σ = 849 MPa

Example of Life Estimation with ε-N Approach

PART (b): Find notch stress and strain after unloading from the monotonic load in part (a) to zero

- For unloading, we can assume Masing behavior with a factor of two expansion of the monotonic stress-strain curve.
- Unloading is from $S_1 = S_{max}$ = 600 MPa to S_2 = 0, or ΔS = 600 MPa.
- From **Neuber's rule** we obtain:

$$\Delta\varepsilon\,\Delta\sigma = \frac{\left(K_f\,\Delta S\right)^2}{E} = \frac{\left(3x600\right)^2}{207000} = 15.65$$

$$\Delta\varepsilon = \frac{\Delta\sigma}{E} + 2\left(\frac{\Delta\sigma}{2K}\right)^{1/n} = \frac{\Delta\sigma}{207000} + 2\left(\frac{\Delta\sigma}{2344}\right)^{1/0.06}$$

resulting in: $\Delta\sigma$ = 1566 MPa,
and $\Delta\varepsilon$ = 0.0100.

- Residual stress, σ_{min}, and strain, ε_{min}, after unloading are calculated as (and shown in Fig. 7.23) :

  $\sigma_{min} = \sigma_{max} - \Delta\sigma$ = 903 -1566 = -663 MPa, and

  $\varepsilon_{min} = \varepsilon_{max} - \Delta\varepsilon$ = 0.0173 - 0.0100 = 0.0073.

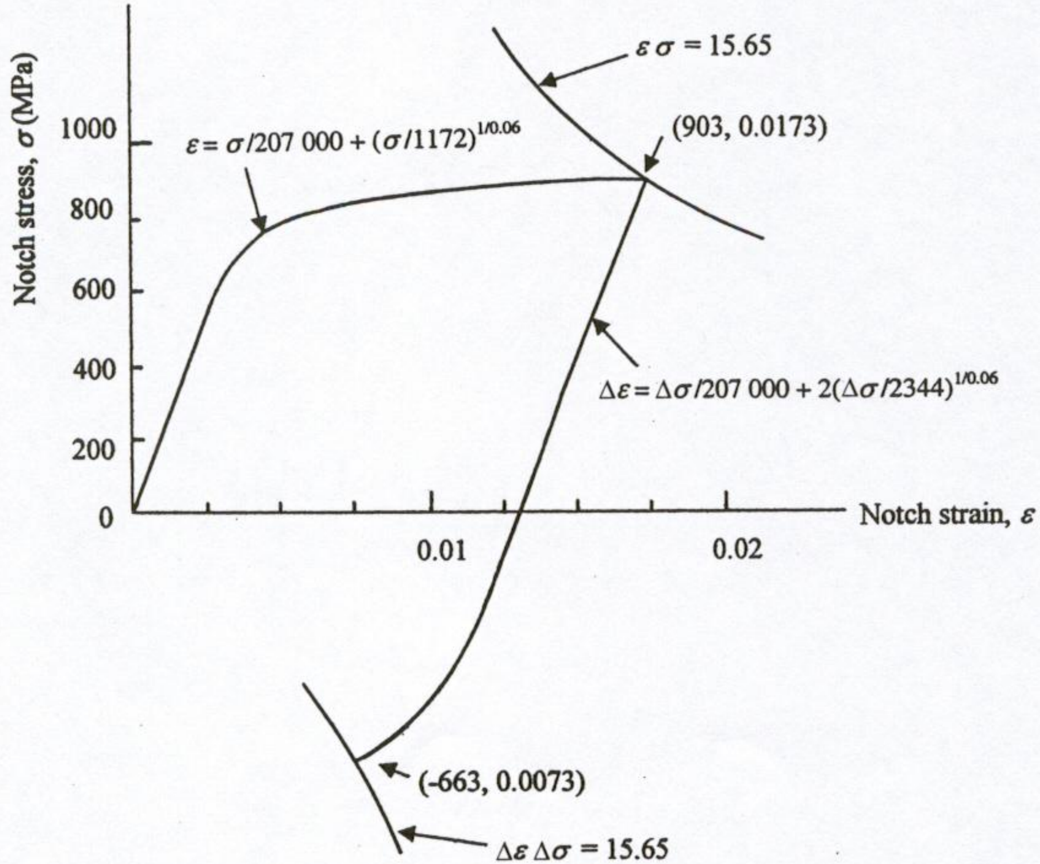


Figure 7.23 Determination of the strain at the root of a notch according to Neuber's rule; material: RQC-100; nominal stress 600 MPa followed by unloading to zero; $K_t = 3$.

Example of Life Estimation with ε-N Approach

- Using the **strain energy density** rule:

$$\frac{(\Delta\sigma)^2}{E} + \frac{4\Delta\sigma}{n+1}\left(\frac{\Delta\sigma}{2K}\right)^{1/n} = \frac{(K_t\,\Delta S)^2}{E}$$

Or $$\frac{(\Delta\sigma)^2}{207000} + \frac{4\Delta\sigma}{0.06+1}\left(\frac{\Delta\sigma}{2x1172}\right)^{1/0.06} = \frac{(3x600)^2}{207000} = 15.65$$

resulting in $\Delta\sigma$ = 1524 MPa, and

$$\Delta\varepsilon = \frac{\Delta\sigma}{E} + 2\left(\frac{\Delta\sigma}{2K}\right)^{1/n} = \frac{1524}{207000} + 2\left(\frac{1524}{2344}\right)^{1/0.06} = \underline{0.0089}$$

Therefore, $\sigma_{min} = \sigma_{max} - \Delta\sigma$ = 872 - 1524 = <u>-652 MPa</u>, and

$\varepsilon_{min} = \varepsilon_{max} - \Delta\varepsilon$ = 0.0115 - 0.0089 = <u>0.0026</u>.

Example of Life Estimation with ε-N Approach

- The **linear rule** does not result in a notch strain after unloading. Notch stress after unloading is calculated as:

$$0.0087 = \frac{\Delta\sigma}{207000} + 2\left(\frac{\Delta\sigma}{2344}\right)^{1/0.06}$$

or $\Delta\sigma$ = 1515 MPa

Therefore: $\sigma_{min} = \sigma_{max} - \Delta\sigma$ = 849 - 1515 = -666 MPa.

Example of Life Estimation with ε-N Approach

Part (c): Find notch stress and strain amplitudes from constant amplitude alternating loads between 4.45 kN and 44.5 kN.

$S_{max} = 11.25\ P_{max} = 11.25\ (44.5) = 500$ MPa

$S_{min} = 11.25\ P_{min} = 11.25\ (4.45) = 50$ MPa

$S_a = (S_{max} - S_{min})/2 = (500 - 50)/2 = 225$ MPa

The nominal behavior for cyclic loading is also elastic since S_{max}, $|S_{min}|$, and S_a are smaller than $S'_y = 600$ MPa.

Example of Life Estimation with ε-N Approach

- We first need to calculate notch root stress and strain at the maximum load.

- Using **Neuber's rule** we use the fatigue notch factor, $K_f = 2.82$, which was calculated in Example 7.2.4:

$$\varepsilon_{max}\,\sigma_{max} = \frac{(K_f S_{max})^2}{E} = \frac{(2.82\,x\,500)^2}{207000} = 9.6$$

$$\varepsilon_{max} = \frac{\sigma_{max}}{E} + \left(\frac{\sigma_{max}}{K'}\right)^{1/n'} = \frac{\sigma_{max}}{207000} + \left(\frac{\sigma_{max}}{1434}\right)^{1/0.14}$$

This results in σ_{max} = 745 MPa, and ε_{max} = 0.0129.

Example of Life Estimation with ε-N Approach

- **Unloading** takes place from 500 MPa to 50 MPa, or ΔS = 450 MPa. Therefore, from **Neuber's rule** we obtain:

$$\Delta\varepsilon\,\Delta\sigma = \frac{(K_f\,\Delta S)^2}{E} = \frac{(2.82\,x\,450)^2}{207000} = 7.78$$

$$\Delta\varepsilon = \frac{\Delta\sigma}{E} + 2\left(\frac{\Delta\sigma}{2K'}\right)^{1/n'} = \frac{\Delta\sigma}{207000} + 2\left(\frac{\Delta\sigma}{2868}\right)^{1/0.14}$$

resulting in $\Delta\sigma$ = 1082 MPa, and $\Delta\varepsilon$ = 0.0072.

Therefore, $\sigma_{min} = \sigma_{max} - \Delta\sigma$ = 745 - 1082 = -337 MPa,

$\sigma_m = (\sigma_{max} + \sigma_{min})/2$ = (745 - 337)/2 = 204 MPa

and $\varepsilon_a = \Delta\varepsilon/2$ = 0.0072/2 = 0.0036,

$\sigma_a = \Delta\sigma/2$ = 1082/2 = 541 MPa.

Example of Life Estimation with ε-N Approach

- Using the **strain energy density** rule, at maximum load:

$$\frac{\sigma^2_{\max}}{E} + \frac{2\sigma_{\max}}{n'+1}\left(\frac{\sigma_{\max}}{K'}\right)^{1/n'} = \frac{(K_t S_{\max})^2}{E}$$

Or

$$\frac{\sigma^2_{\max}}{207000} + \frac{2\sigma_{\max}}{0.14+1}\left(\frac{\sigma_{\max}}{1434}\right)^{1/0.14} = \frac{(3x500)^2}{207000} = 10.9$$

which gives σ_{max} = 712 MPa.

Substituting this value into the cyclic stress-strain equation results in:

$$\varepsilon_{\max} = \frac{\sigma_{\max}}{E} + \left(\frac{\sigma_{\max}}{K'}\right)^{1/n'} = \frac{712}{207000} + \left(\frac{712}{1434}\right)^{1/0.14} = 0.0102$$

Example of Life Estimation with ε-N Approach

- For **unloading**: $$\frac{(\Delta\sigma)^2}{E} + \frac{4\Delta\sigma}{n'+1}\left(\frac{\Delta\sigma}{2K'}\right)^{1/n'} = \frac{(K_t\,\Delta S)^2}{E}$$

or $$\frac{(\Delta\sigma)^2}{207000} + \frac{4\Delta\sigma}{0.14+1}\left(\frac{\Delta\sigma}{2x1434}\right)^{1/0.14} = \frac{(3x450)^2}{207000} = 8.8$$

resulting in $\Delta\sigma$ = 1070 MPa, and

$$\Delta\varepsilon = \frac{\Delta\sigma}{E} + 2\left(\frac{\Delta\sigma}{2K'}\right)^{1/n'} = \frac{1070}{207000} + 2\left(\frac{1070}{2868}\right)^{1/0.14} = 0.0069$$

Therefore, $\sigma_{min} = \sigma_{max} - \Delta\sigma$ = 712 - 1070 = -358 MPa,

$\sigma_m = (\sigma_{max} + \sigma_{min})/2$ = (712 - 358)/2 = 177 MPa

and $\varepsilon_a = \Delta\varepsilon/2$ = 0.0069/2 = 0.0035,

$\sigma_a = \Delta\sigma/2$ = 1070/2 = 535 MPa.

Example of Life Estimation with ε-N Approach

- If we use the **linear rule**:

$$\varepsilon_{\max} = \frac{K_t S_{\max}}{E} = \frac{3x500}{207000} = 0.0072$$

$$0.0072 = \frac{\sigma_{\max}}{207000} + \left(\frac{\sigma_{\max}}{1434}\right)^{1/0.14}$$ or σ_{max} = 663 MPa

Then

$$\Delta\varepsilon = K_t \Delta e = \frac{K_t \Delta S}{E} = \frac{3x450}{207000} = 0.0065$$

$$0.0065 = \frac{\Delta\sigma}{207000} + 2\left(\frac{\Delta\sigma}{2868}\right)^{1/0.14}$$ or $\Delta\sigma$ = 1045 MPa

Therefore, $\sigma_{min} = \sigma_{max} - \Delta\sigma$ = 663 - 1045 = -382 MPa,

$\sigma_m = (\sigma_{max} + \sigma_{min})/2$ = (663 – 382)/2 = <u>141 MPa</u>

and $\varepsilon_a = \Delta\varepsilon/2$ = 0.0065/2 = <u>0.0033</u>,

$\sigma_a = \Delta\sigma/2$ = 1045/2 = <u>523 MPa</u>.

Example of Life Estimation with ε-N Approach

PART (d): **Find the expected fatigue life to the formation of cracks on the order of 1 mm from the loading in part (c)**

We can use the Smith-Watson-Topper mean stress parameter, or Morrow's mean stress parameters from Chapter 5. Here we choose the Smith-Watson-Topper parameter:

$$\varepsilon_a \sigma_{max} = \frac{\left(\sigma'_f\right)^2 \left(2N_f\right)^{2b}}{E} + \varepsilon'_f \sigma'_f \left(2N_f\right)^{b+c} = 7.43 \left(2N_f\right)^{-0.14} + 818 \left(2N_f\right)^{-0.76}$$

Results Are Summarized as Follows

	Neuber's Rule	Strain Energy Density Rule	Linear Rule
(*a*) Monotonic loading to 53.4 kN			
Notch stress, MPa	903	872	849
Notch strain	0.0173	0.0115	0.0087
(*b*) Unloading from 53.4 kN to 0			
Notch residual stress, MPa	−663	−652	−666
Notch strain	0.0073	0.0026	0
(*c*) Cyclic loading between 4.45 and 44.5 kN			
Notch stress amplitude, MPa	541	535	523
Notch strain amplitude	0.0036	0.0035	0.0033
(*d*) Fatigue life for loading in (*c*)			
Notch mean stress, σ_m (MPa)	204	177	141
Maximum notch stress, σ_{max} (MPa)	745	712	663
Product of $\varepsilon_a \sigma_{max}$	2.68	2.49	2.16
Fatigue life, N_f (cycles)	5750	7400	12 750

- It can be seen from this table that the most conservative fatigue life is obtained from Neuber's rule and the least conservative life from the linear rule, result from strain energy density rule falling in between.

- If K_t rather than K_f had been used in the application of Neuber's rule for cyclic loading, the notch stress amplitude would be 561 MPa rather than 541 MPa, the notch strain amplitude would be 0.0039 rather than 0.0036, and the expected fatigue life would be 3950 cycles rather than 5750 cycles. Therefore, more conservative predictions are obtaind by using K_t, as expected.

- The notch stress amplitudes obtained from notch stress/strain analysis models presented could also be used in conjunction with the S-N approach. In this case, the S-N line or equation for the smooth behavior should be used, since the effect of the notch is already incorporated in the stress amplitude.

- Since the minimum stress in Part (c) is 50 MPa, the fatigue lives predicted from the strain-life approach in this example can be directly compared to those from the *S-N* approach in Section 7.2.4.

- From Fig. 7.16, for a nominal stress amplitude of S_a = 225 MPa, the predicted life using Basquin's equation for the notched part is about 11000 cycles based on the Haigh diagram approach for long life, and about 28000 cycles based on the modified Goodman equation.

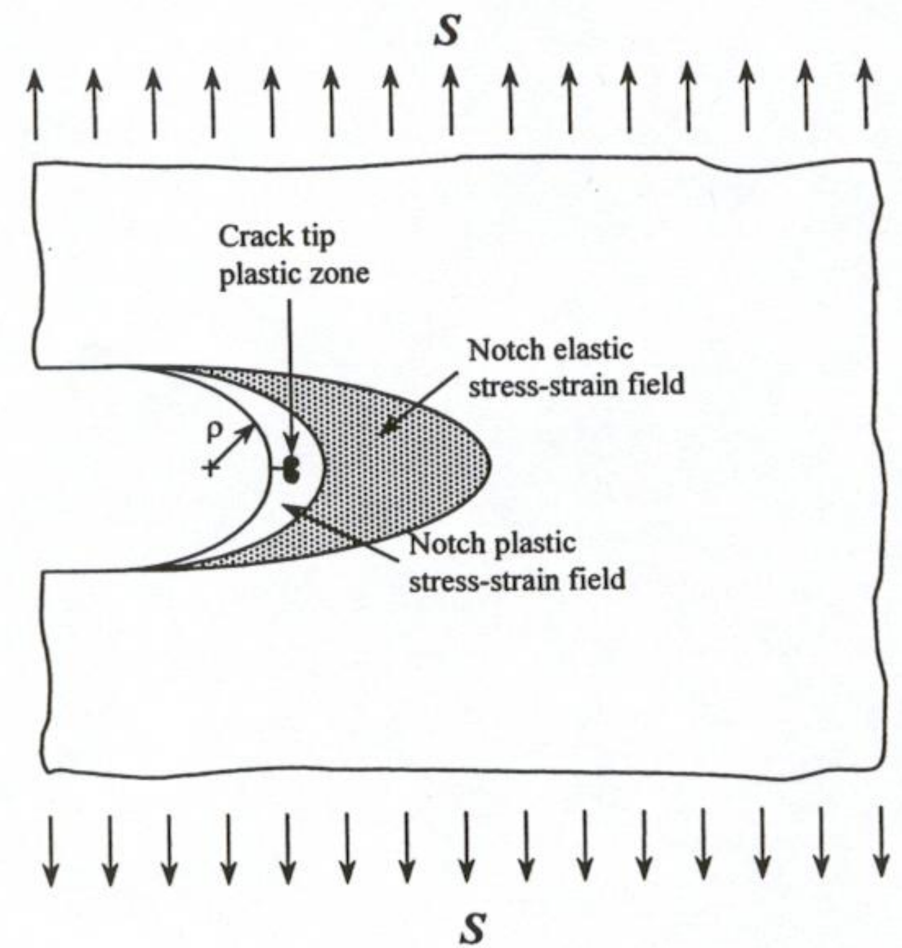
- The remaining life (crack growth life from formation of a crack to fracture) of the notched part in this example problem can be estimated based on linear elastic fracture mechanics discussed in Chapter 6. This is done in the next section.

CRACK GROWTH AT NOTCHES

CRACK GROWTH AT NOTCHES

- Deformation of the material at the notch root under cyclic loading is often inelastic. Thus, cracks often start from a notch under conditions of local plasticity.

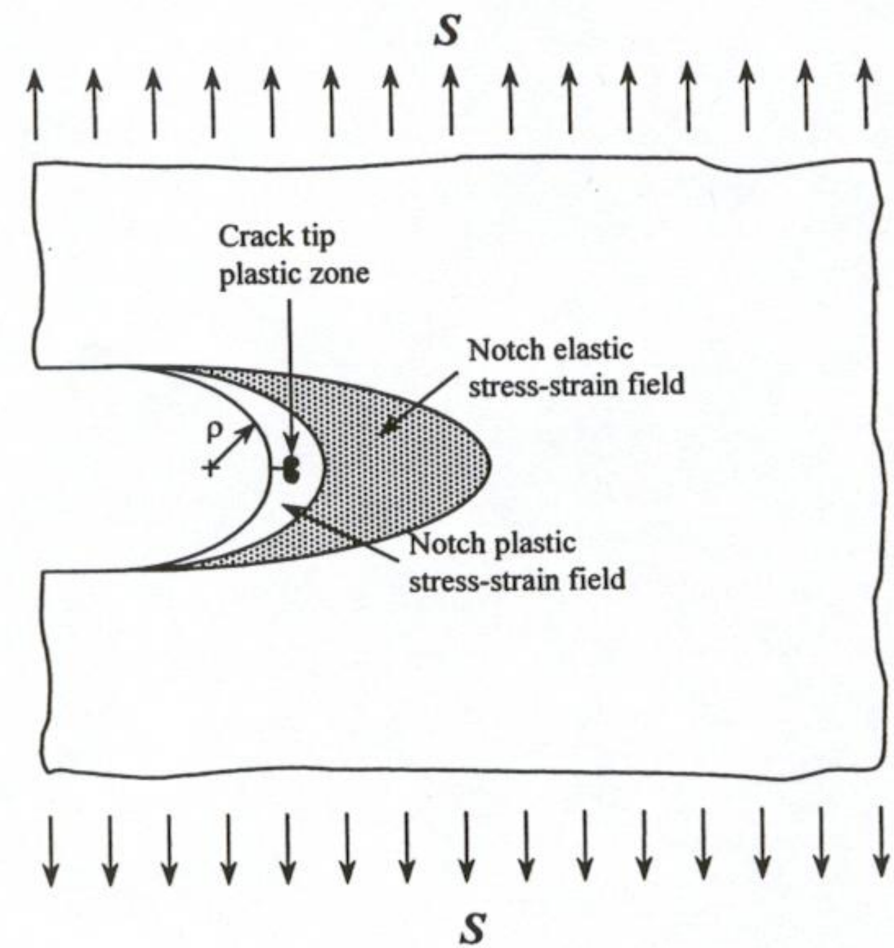
- As the cracks continue to grow, they grow into the elastic stress-strain field of the notch, and with subsequent growth extend into the net section elastic stress-strain field of the specimen or component.



CRACK GROWTH AT NOTCHES

- After the crack exceeds a certain length, usually 10 to 20 percent of the notch radius, the notch geometry has little influence on the elastic stress-strain field of the advancing crack.

- Crack growth through a notch stress-strain field may represent a major or minor portion of the total fatigue life.

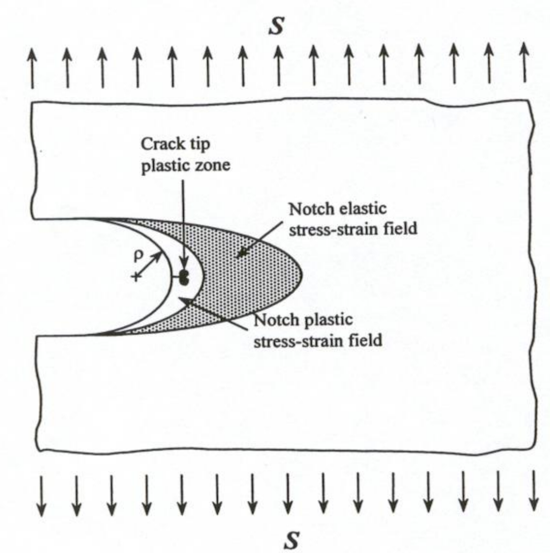


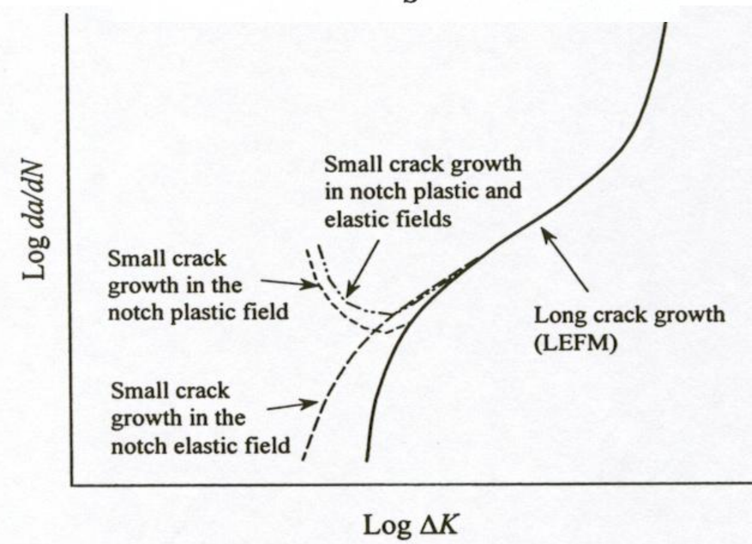
CRACK GROWTH AT NOTCHES

- Because the stress-strain field of a notch varies, FCG behavior also may vary as the crack grows through the different fields, as shown.

- For cracks growing in the **plastic stress-strain field** of a notch, FCG rates are initially high, but decrease as the crack grows further away from the notch root.

- After the crack grows out of the notch plastic zone, its length may no longer be small, and the crack growth behavior merges with the long crack growth curve.

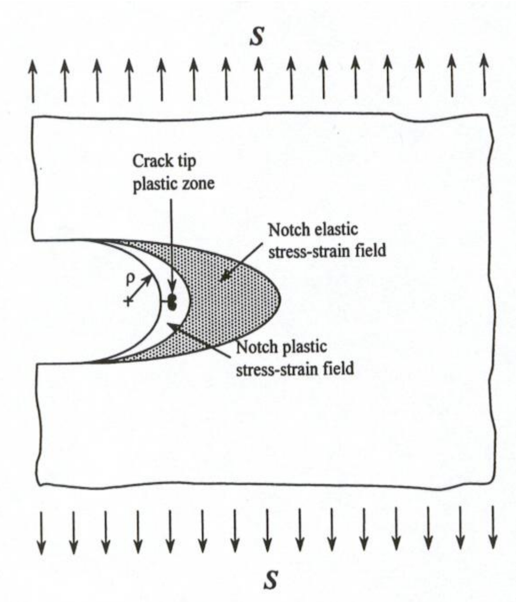


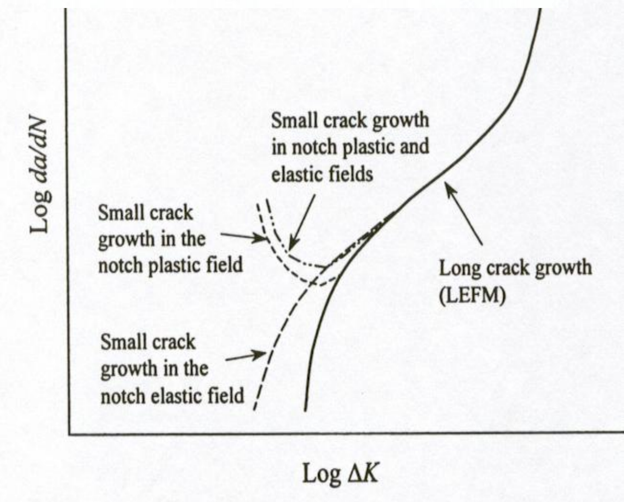


CRACK GROWTH AT NOTCHES

- Crack growth behavior for small cracks growing in the **elastic stress-strain field** of a notch is often similar to that for physically small cracks.

- Crack growth behavior in the **plastic and elastic stress-strain field** of the notch in general is a combination of the previous two conditions, assuming the crack is small, as the crack grows from the plastic to the elastic stress-strain field.





CRACK GROWTH AT NOTCHES

- For a crack that nucleates from a **shallow or blunt notch**, the fatigue behavior is often dominated by **crack nucleation**.

- Conversely, a crack may nucleate from a **sharp notch** rather quickly due to the elevated stresses at the notch root but it may stop growing once the crack grows through the notch-influenced region. For this condition, **crack growth** may dominate the fatigue behavior or total fatigue life.

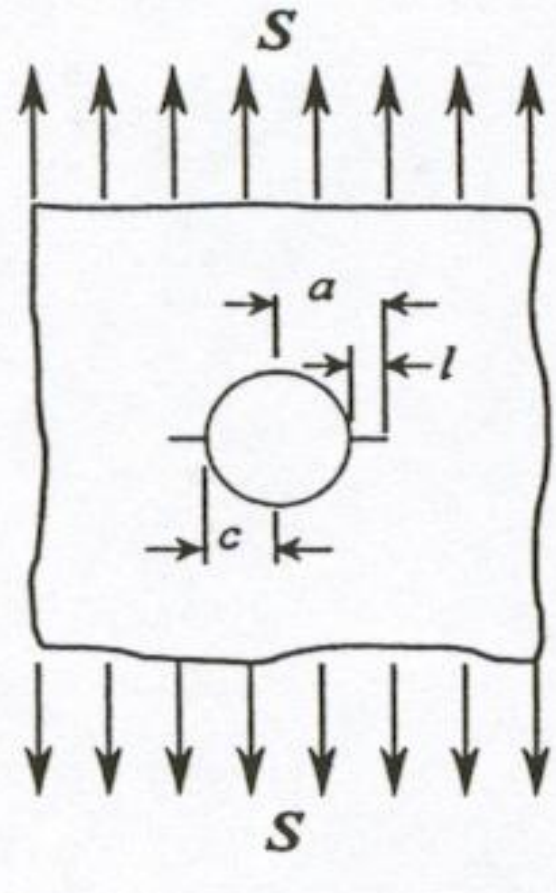
CRACK GROWTH AT NOTCHES

- To quantify the influence of the stress-strain field of a notch on crack growth, consider a **very wide plate** with a circular hole containing a pair of small through-thickness cracks as shown.

- When the length of the fatigue crack, *l*, i small compared to the radius of the circle, *c*, the stress intensity factor, K_{smal} can be approximated by

$$K_{small} \approx 1.12(K_t S)\sqrt{\pi l}$$

where *S* is the net section stress.



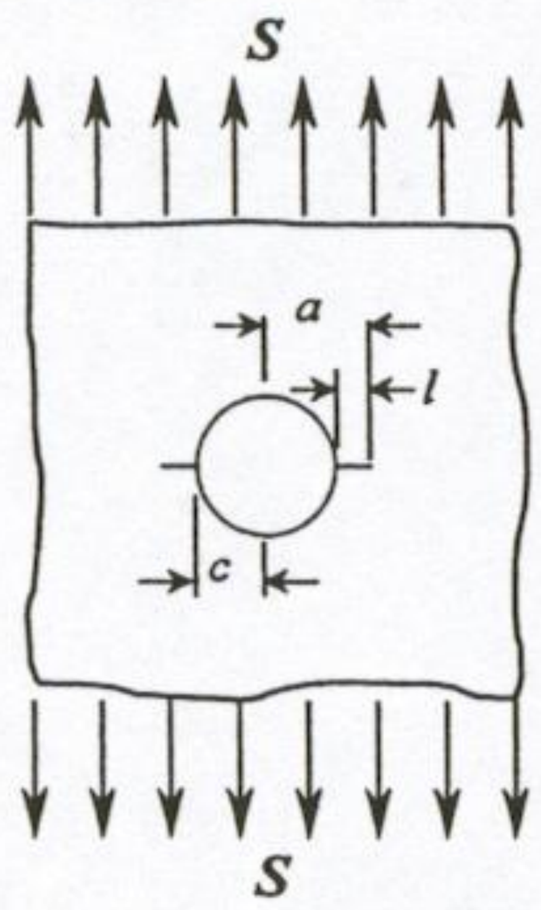
(a)

CRACK GROWTH AT NOTCHES

- Once the crack has grown a sufficient distance from the hole, the stress intensity solution takes on the form:

$$K_{long} \approx S\sqrt{\pi a}$$

where $a = c + l$ and S is based on the gross area. Thus, for long cracks, the width or diameter of the hole acts as part of the crack.

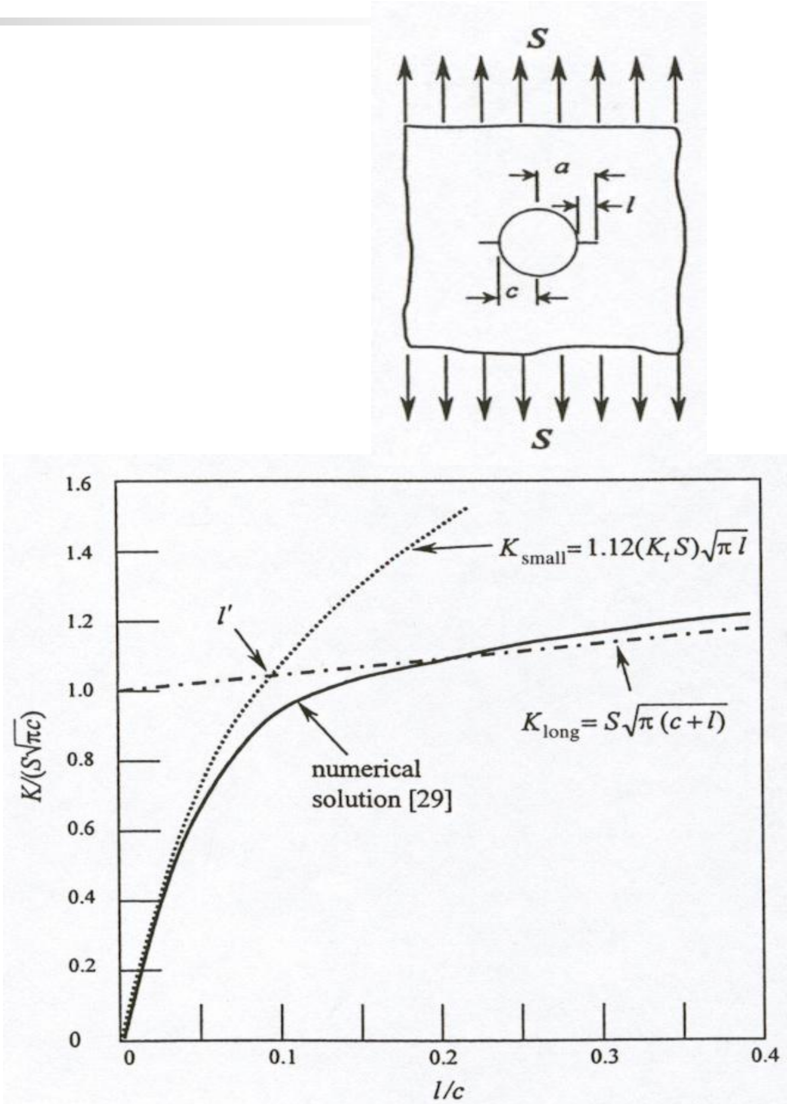


CRACK GROWTH AT NOTCHES

- Figure 7.26*b* shows variation of K as a function of the crack length, *l*, normalized by the radius of the hole, *c*.

- An intersection point can be approximated which identifies a transitional crack length where K_{small} can be used for crack lengths less than l' and K_{long} can be used for crack lengths greater than l' :

$$l' = \frac{c}{(1.12\,K_t)^2 - 1}$$



CRACK GROWTH AT NOTCHES

- The stress intensity factor solutions presented are valid for situations where notch tip plasticity effects are negligible.

- When this is the case, l' is usually a small fraction of the root radius, ρ, of the notch or stress concentration, where usually $0.1\rho \leq l' \leq 0.2\rho$.

- Even for conditions where small scale yielding prevails, FCG rates for cracks emanating from notches tend to be higher than their long crack counterparts, as previously shown.
  - A contributing factor to this behavior is the absence or reduction in tensile residual plastic deformation (resulting from crack tip plasticity) in the wake of the crack because the crack is small.
  - Less residual plastic deformation in the crack wake means less crack closure, a greater ΔK_{eff}, and thus higher FCG rates.

CRACK GROWTH AT NOTCHES

- For small cracks contained within the local elastic stress-strain field of a notch, extrapolation of the Paris equation into region I may provide a reasonable approach to life prediction, the same as that presented in Section 6.8 for small cracks.

- For small cracks contained within the plastic stress-strain field of a notch, elastic-plastic fracture mechanics (EPFM) may provide at least a partial solution to the differences observed between the behavior of small cracks emanating from notches and long cracks.

- However, the decreased driving force with increasing crack length typically observed for these types of cracks is difficult to account for using LEFM or EPFM without use of some type of crack closure model.

- Many attempts have been made to account for the anomalous fatigue crack growth from notches. While these models have proven useful, they have not provided a complete correlation between notch fatigue crack growth data and LEFM long crack growth data.

THE TWO STAGE APPROACH

THE TWO STAGE APPROACH

- The life of a structure is often considered in two stages:
  - Stage 1: Life to the formation of a crack that is on the order of 1 mm, “crack nucleation”.
  - Stage 2: Life from this existing crack to failure, “crack growth.”

- In some situations,
  - stage 2 may be negligible in comparison to stage 1 (i.e. rotating bending of a small notched shaft)
  - on the other hand, cracks may be present. Stage 1 then becomes negligible in comparison to stage 2.

- Predictions of stage 1 are most commonly done using the ε-N method. Predictions of stage 2 are done using LEFM.

EXAMPLE OF THE TWO STAGE APPROACH

- In Section 7.3.5*d*, fatigue life calculations were made for stage 1 using the strain-life approach.
- We can predict the fatigue life associated with stage 2 (fatigue crack growth).
- The "total fatigue life" includes both fatigue crack nucleation life with a crack on the order of 1 mm (stage 1) and fatigue crack growth life (stage 2).

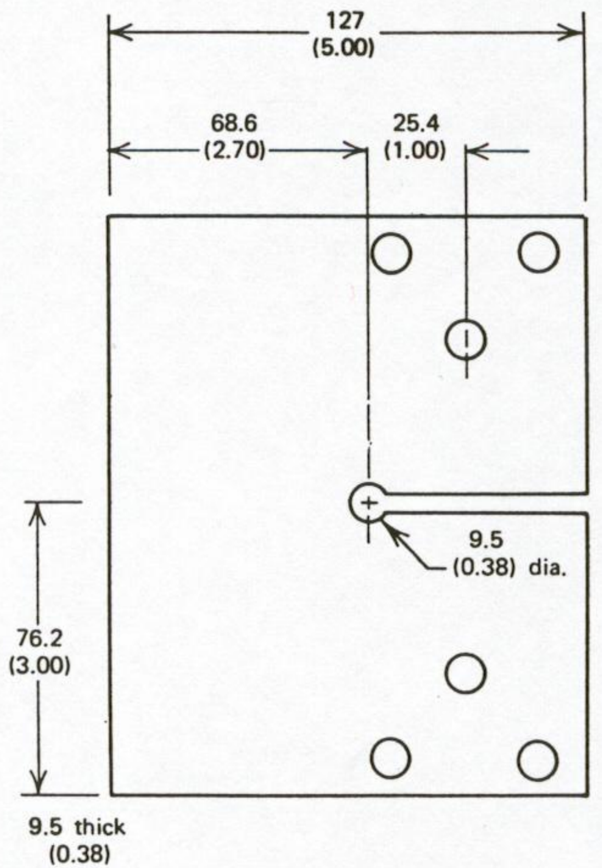


Figure 7.13 Test specimen design, mm (in.).

EXAMPLE OF THE TWO STAGE APPROACH

- To use LEFM for Stage 2 Fatigue Life Calculations, the stress intensity factor solution for the specimen geometry is required, which is given by [33]:

$$K = \frac{(0.247)P}{\sqrt{w}}\left[-54.98 + 417.28\left(\frac{a}{w}\right) - 939.76\left(\frac{a}{w}\right)^2 + 739.31\left(\frac{a}{w}\right)^3\right]$$

where *P* is in kN and *K* is in MPa√m.

This expression is applicable for a specimen thickness *t* equal to 9.5 mm and *a/w* between about 0.32 and 0.75, which corresponds to crack lengths *a* as measured from the load line of 30 to 70 mm.

EXAMPLE OF THE TWO STAGE APPROACH

- The fracture toughness, K_c, for RQC-100 for this thickness, along with the Paris equation for $R \approx 0$ are:

$K_c \approx 165$ MPa√m and $$\frac{da}{dN} = 2.80\,x10^{-12}(\Delta K)^{3.25}$$

Although this crack growth rate equation was developed for $R = 0$, the mean stress effect ($R = 0.1$) in this example problem will not have much effect on crack growth compared to $R = 0$.

EXAMPLE OF THE TWO STAGE APPROACH

- The strain-life prediction model is for crack nucleation lengths on the order of 1 mm.

- The LEFM model must then assume an initial crack length comparable with this length, thus let the initial crack length be 1 mm.

- We should check to see if this crack size emanating from the keyhole notch is greater than the transitional crack length l'.

- Although the expression in Eq. 7.33 for transitional crack length is for a crack emanating from both sides of a hole in a very wide plate, it can be used as a first approximation for the transitional crack length in this example problem (A more precise solution would take into account the geometry factor)

$$l' = \frac{c}{(1.12 K_t)^2 - 1} = \frac{9.5/2}{[1.12(3)]^2 - 1} = 0.46\ mm \approx 0.5\ mm$$

EXAMPLE OF THE TWO STAGE APPROACH

- Therefore, the crack length used to predict the fatigue life for stage 1 (1 mm) is greater than the estimated transitional crack length and can be assumed to be outside the influence of the notch stress-strain field.

- Thus the long crack stress intensity factor solution can be used:

$$K = \frac{(0.247)P}{\sqrt{w}}\left[-54.98 + 417.28\left(\frac{a}{w}\right) - 939.76\left(\frac{a}{w}\right)^2 + 739.31\left(\frac{a}{w}\right)^3\right]$$

EXAMPLE OF THE TWO STAGE APPROACH

- The initial crack length is 25.4 mm + (9.5mm)/2 = 30.15 mm plus 1 mm, a_i = 31.15 mm.
- Initial $a/w = 0.33$.
- Initial ΔK for this length crack with $\Delta P = P_{max} - P_{min} = 44.5 - 4.45 \approx$ 40 kN is 69 MPa$\sqrt{}$m.
- This value is well above ΔK_{th} levels and thus the Paris equation is applicable from that standpoint.

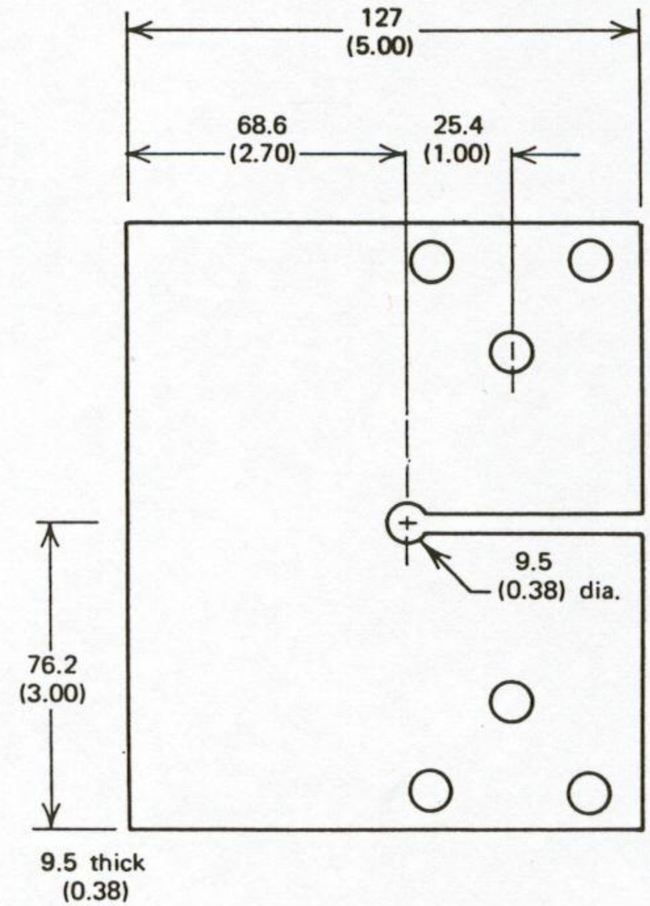


Figure 7.13 Test specimen design, mm (in.).

EXAMPLE OF THE TWO STAGE APPROACH

- Calculation of the plane stress initial plastic zone radius $2r_y$ using K_{max} = 77 MPa$\sqrt{m}$ and S_y = 883 MPa gives $2r_y$ = 2.4 mm, or r_y = 1.2 mm:

$$2r_y = \frac{1}{\pi}\left(\frac{K_{max}}{S_y}\right)^2$$

- While the initial ΔK is approaching region III of the sigmoidal curve ($da/dN \approx$ 2.7 x 10^{-6} m/cycle), we use the LEFM model and the Paris equation to obtain a first approximation.

- The critical crack length at fracture can be approximated by setting K equal to K_c and P= P_{max} = 44.5 kN, resulting in a_c = 54 mm.

EXAMPLE OF THE TWO STAGE APPROACH

- The fatigue crack growth life, N_f, which we define here as N_g can be found by integrating the Paris equation:

$$\frac{da}{dN} = A(\Delta K)^n = A(K_{max} - K_{min})^n$$

or

$$N_g = N_f = \int_0^{N_f} dN = \int_{a_i}^{a_c} \frac{da}{A(K_{max} - K_{min})^n} =$$

$$\frac{1}{2.8x10^{-12}\left(\blacksquare.247(P_{max} - P_{min})\right)^{3.25}} \int_{a_i}^{a_c} \frac{da}{\left[-54.98 + 417.28\left(\frac{a}{w}\right) - 939.76\left(\frac{a}{w}\right)^2 + 739.31\left(\frac{a}{w}\right)^3\right]^{3.25}}$$

resulting in $N_g \approx 3000$ cycles

EXAMPLE OF THE TWO STAGE APPROACH

- This predicted fatigue crack growth life is 1/2 of the fatigue life calculated for stage 1, N_n, using Neuber's rule, and a smaller fraction compared to the strain energy density or the linear rule.

- Thus, for this example, stage 1 has a greater contribution to the total fatigue life, N_t, where $N_t = N_n + N_g$.

- Now let us assume a keyhole diameter of 5 mm instead of 9.5 mm, which results in $K_t = 4.1$.
  - Using Neuber's rule and the Smith-Watson-Topper parameter, the fatigue life to approximately 1 mm results in $N_n \approx 1500$ cycles.
  - The fatigue crack growth life, N_g, will not change (~3000 cycles) as the initial and final crack lengths do not change.

EXAMPLE OF THE TWO STAGE APPROACH

TABLE 7.2 Fatigue Lives Based on Different Notch Radii and Load Ranges

	$K_t = 3$ ($d = 9.5$ mm)			$K_t = 4.1$ ($d = 5$ mm)		
ΔP (kN)	N_n	N_g	N_t	N_n	N_g	N_t
40	6 000	3 000	9 000	1 500	3 000	4 500
27	111 000	11 000	122 000	13 000	11 000	24 000

- Fatigue life for both crack nucleation and crack growth is given in Table 7.2 for the two notch sizes described above and for two load ranges.
  - For the more blunt notch (smaller K_t) the nucleation life is a larger portion of the total fatigue life. This difference becomes even greater for the smaller load range.
  - For the sharper notch (larger K_t) the crack growth life is roughly 2 times longer than the nucleation life for the higher load range, while the nucleation life and crack growth life are similar for the lower load range.

EXAMPLE OF THE TWO STAGE APPROACH

- Thus, the fatigue crack nucleation and fatigue crack growth lives are dependent on both the notch geometry and load level.

- Fatigue crack nucleation life dominates the total fatigue life at low load and the 9.5 mm diameter hole, while the fatigue crack growth life is a greater percentage of the total fatigue life at the high load and the 5 mm diameter hole.

SUMMARY AND DOS AND DON'TS IN DESIGN

- Notches cannot be avoided in many structures and machines and understanding their effect is of key importance in fatigue.
- Fatigue strength of notched members is not only affected by the stress concentration factor, but also depends on the stress gradient, mean or residual stress at the notch, local yielding, and development and growth of cracks.
- The ratio of smooth to notched fatigue strength is called the fatigue notch factor, K_f. This factor depends on the stress concentration fator, K_t, notch sensitivity of the material, q, and the mean and alternating stress levels.
- For zero mean stress and long life, $K_f = 1 + q\,(K_t - 1)$
  where empirical formulas for q have been developed by Neuber and Peterson.

SUMMARY AND DOS AND DON'TS IN DESIGN

- In the *S*-N approach for fully reversed fatigue behavior of notched members, the *S*-*N* line can be approximated by connecting the ultimate tensile strength at 1 cycle to the notched fatigue strength (S_f/K_f) at long life.

- If a mean stress exists, fatigue strength of the notched member at long life is estimated from a Haigh diagram or by using the modified Goodman equation:

$$\frac{S_a}{(S_f/K_f)} + \frac{S_m}{S_u} = 1$$

SUMMARY AND DOS AND DON'TS IN DESIGN

- The strain-life approach for fatigue of notched members involves two steps:
  - First, notch root stresses and strains are found by experimental, finite element, or analytical models.
  - Life prediction is then made using the notch root stresses and strains based on the strain-life equations.

- Analytical models of notch stress/strain include
  - the linear rule,
  - Neuber's rule, and
  - the strain energy density or Glinka's rule.

SUMMARY AND DOS AND DON'TS IN DESIGN

- The **linear rule** assumes the strain concentration factor, K_ε, to be equal to the theoretical stress concentration factor, K_t, which for nominal elastic behavior results in:

$$\Delta \varepsilon = K_t \frac{\Delta S}{E}$$

- This rule agrees with measurements in plane strain situations.

SUMMARY AND DOS AND DON'TS IN DESIGN

- **Neuber's rule** has been the most widely used model. It assumes the geometrical mean of the stress and strain concentration factors to be equal to the theoretical stress concentration factor ($\sqrt{K_\sigma K_\varepsilon} = K_t$).

- This rule generally agrees with measurements in plane stress situations and is more conservative than the linear rule.

- To reduce the degree of conservatism with Neuber's rule, the fatigue notch factor, K_f, is often used in place of K_t.

- For nominal elastic behavior, Neuber's rule can be expressed as:

$$\frac{(\Delta\sigma)^2}{E} + 2\,\Delta\sigma\left(\frac{\Delta\sigma}{2K'}\right)^{1/n'} = \frac{(K_f\,\Delta S)^2}{E}$$

SUMMARY AND DOS AND DON'TS IN DESIGN

- The **strain energy density** or **Glinka's rule** is based on the assumption that the strain energy density at the notch root is nearly the same for linear elastic and elastic-plastic notch behaviors.

- For nominal elastic behavior Glinla's rule is expressed as:

$$\frac{(\Delta\sigma)^2}{E} + \frac{4\Delta\sigma}{n'+1}\left(\frac{\Delta\sigma}{2K'}\right)^{1/n'} = \frac{(K_t\,\Delta S)^2}{E}$$

SUMMARY AND DOS AND DON'TS IN DESIGN

- The local stress-strain field of a notch has a large influence on crack growth behavior.
- For cracks that nucleate from a shallow or blunt notch the fatigue behavior is often dominated by crack nucleation.
- Cracks that nucleate from a sharp notch often nucleate rather quickly due to the elevated local stresses and crack growth often dominates the fatigue behavior for this case.
- The two stage approach can be used to estimate the total fatigue life that includes both crack nucleation (stage 1) and crack growth (stage 2).
  - Strain-life (ε-N) methods are used for stage 1,
  - while stage 2 uses fracture mechanics and its application to FCG.

SUMMARY AND DOS AND DON'TS IN DESIGN

- Do recognize the theoretical stress concentration factor, K_t, only depends on geometry and mode of loading, but that it can only be used to relate notch stress to nominal stress for linear elastic notch deformation behavior.
- Do consider the effects of stress state and stress gradients at notches.
- Do recognize that fatigue strength of notched parts not only depends on the part geometry and loading, but also on material notch sensitivity. The stronger the material, the higher the notch sensitivity.
- Do expect mean stress to have more effect on fatigue behavior of notched parts than in smooth parts.

SUMMARY AND DOS AND DON'TS IN DESIGN

- Do recognize that when significant notch plastic deformation exists, the strain-life approach to life prediction is usually a superior approach to the *S-N* approach.

- Do recognize that fatigue crack growth from a notch may represent a major or minor portion of the total fatigue life.