Example

A notched aluminum component consists of a plate 12 in. wide and 1 in. thick with a 2 in. diameter center hole. Determine the crack initiation life of the component under the axial stress histories shown below. History A is a constant amplitude zero-to-maximum (R=0) axial stress. Histories B and C have initial overloads followed by the same constant amplitude zero-to-maximum stress fluctuations.

Geometry; W = 12 in.   t = 1 in.   d = 2.in, K_t = 2.56

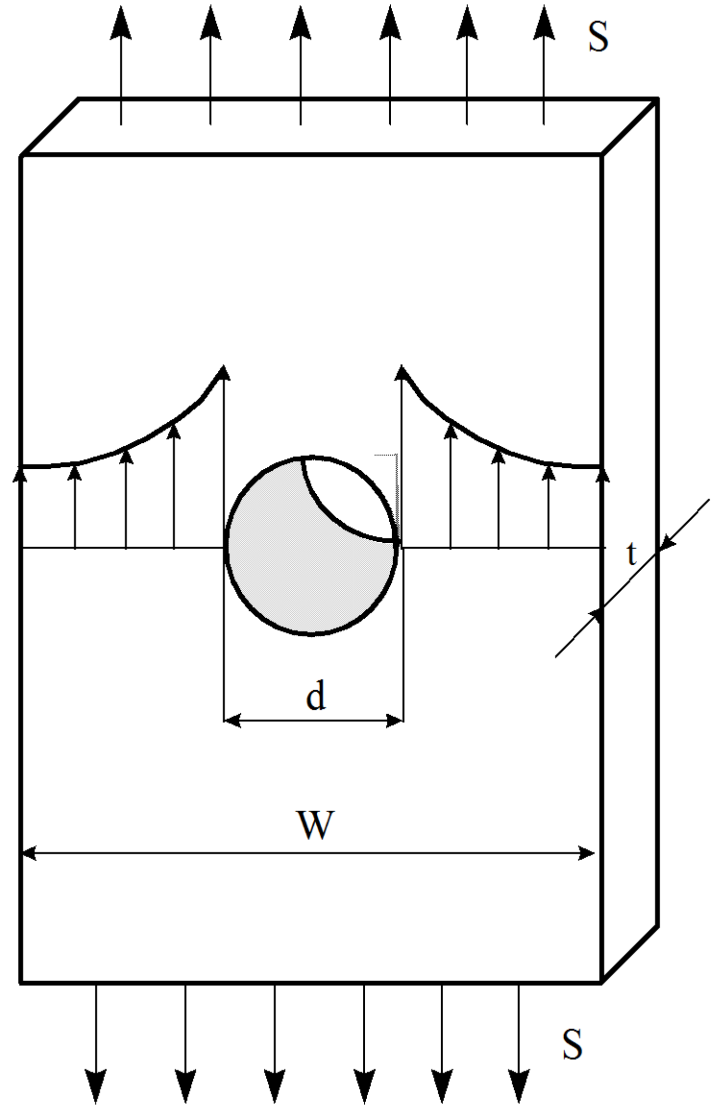
Material;

Cyclic stress – strain curve:

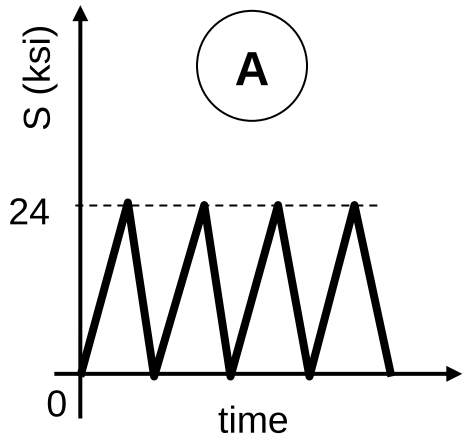
E = 10600 ksi, K' = 95 ksi, n' = 0.065

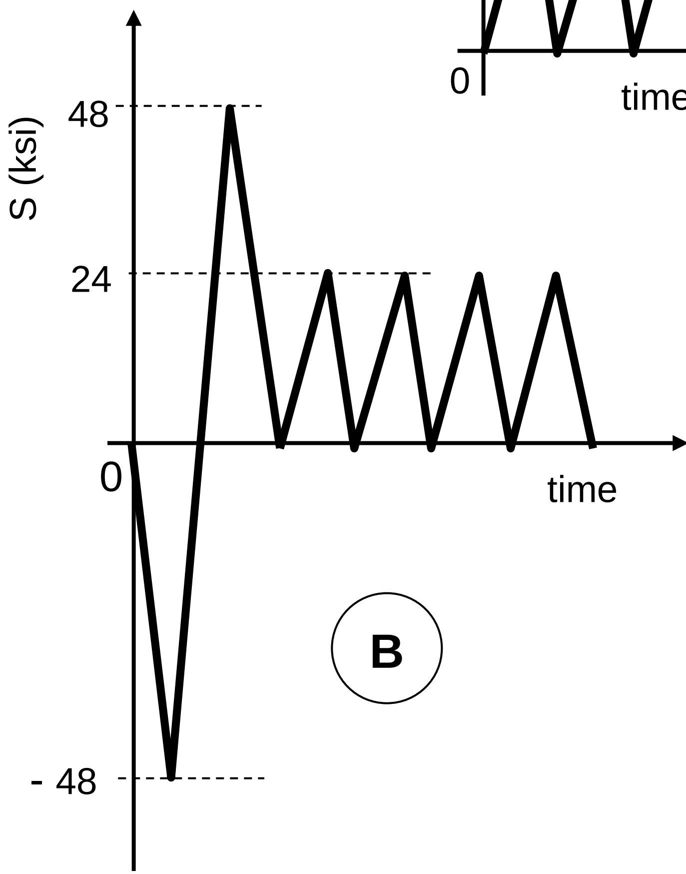
Fatigue strain-life curve:

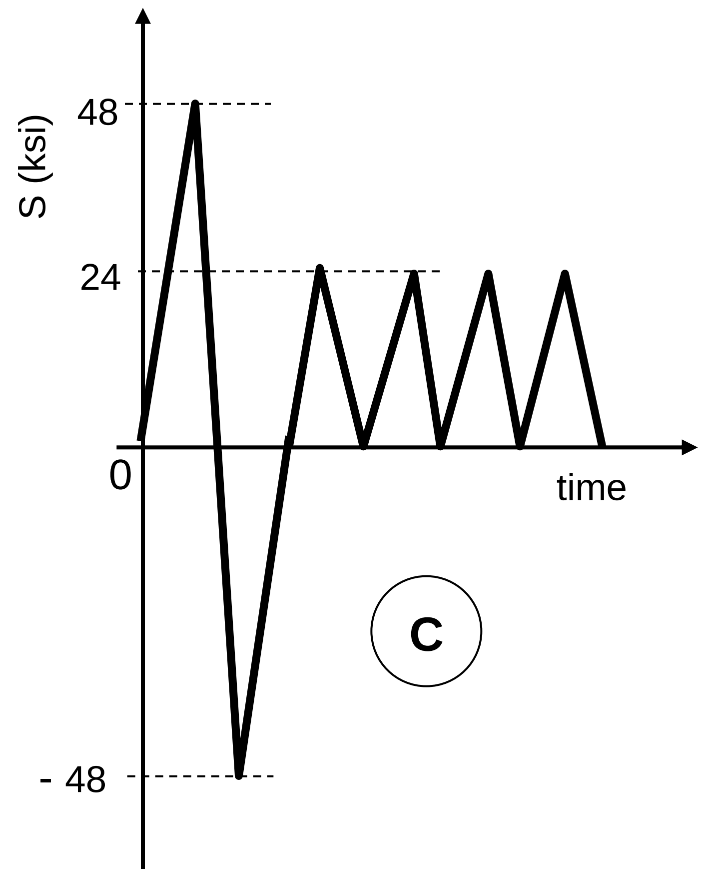
σ_f' = 160 ksi, b = -0.124, ε_f' = 0.22, c = -0.59

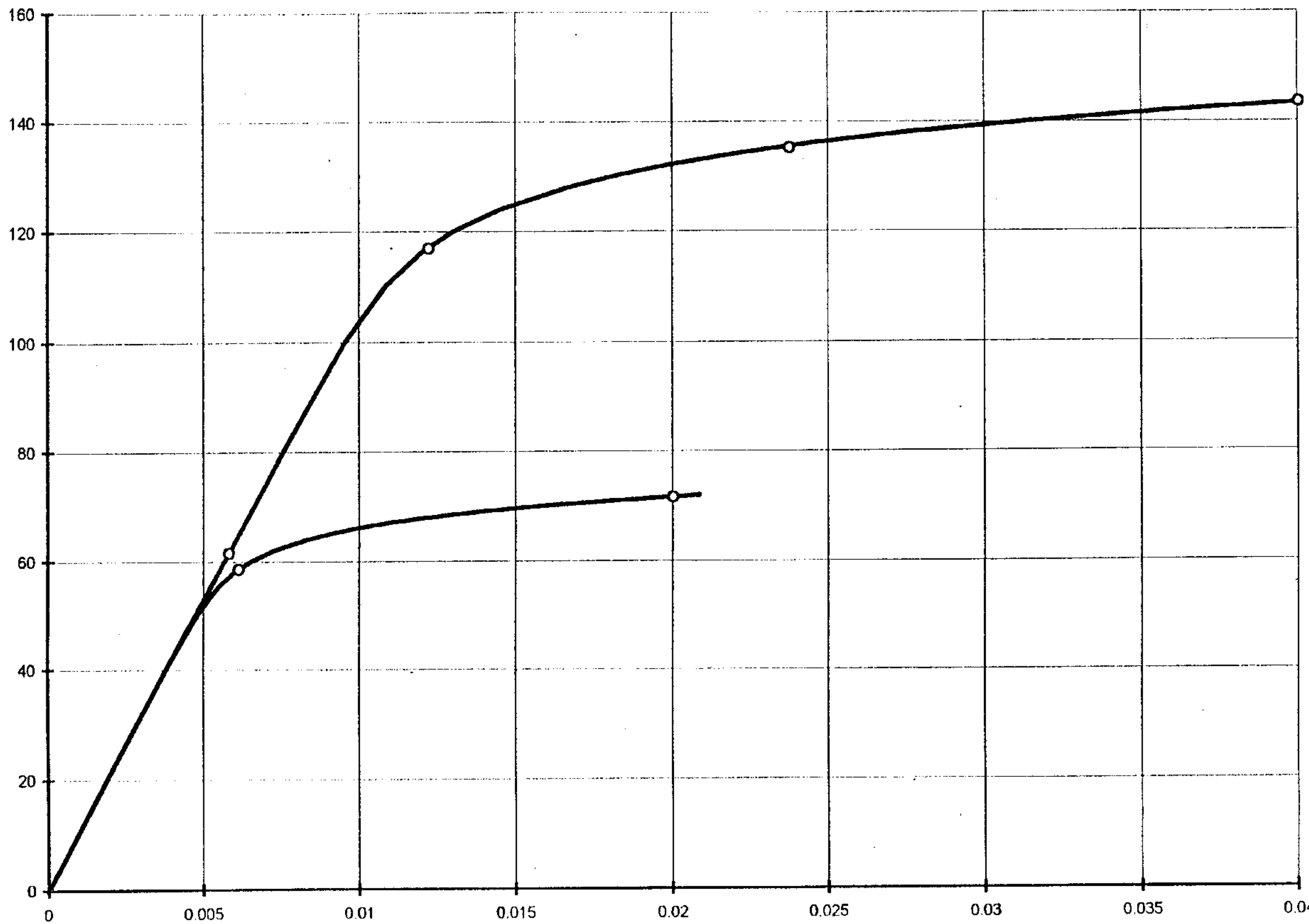


Nominal stress histories

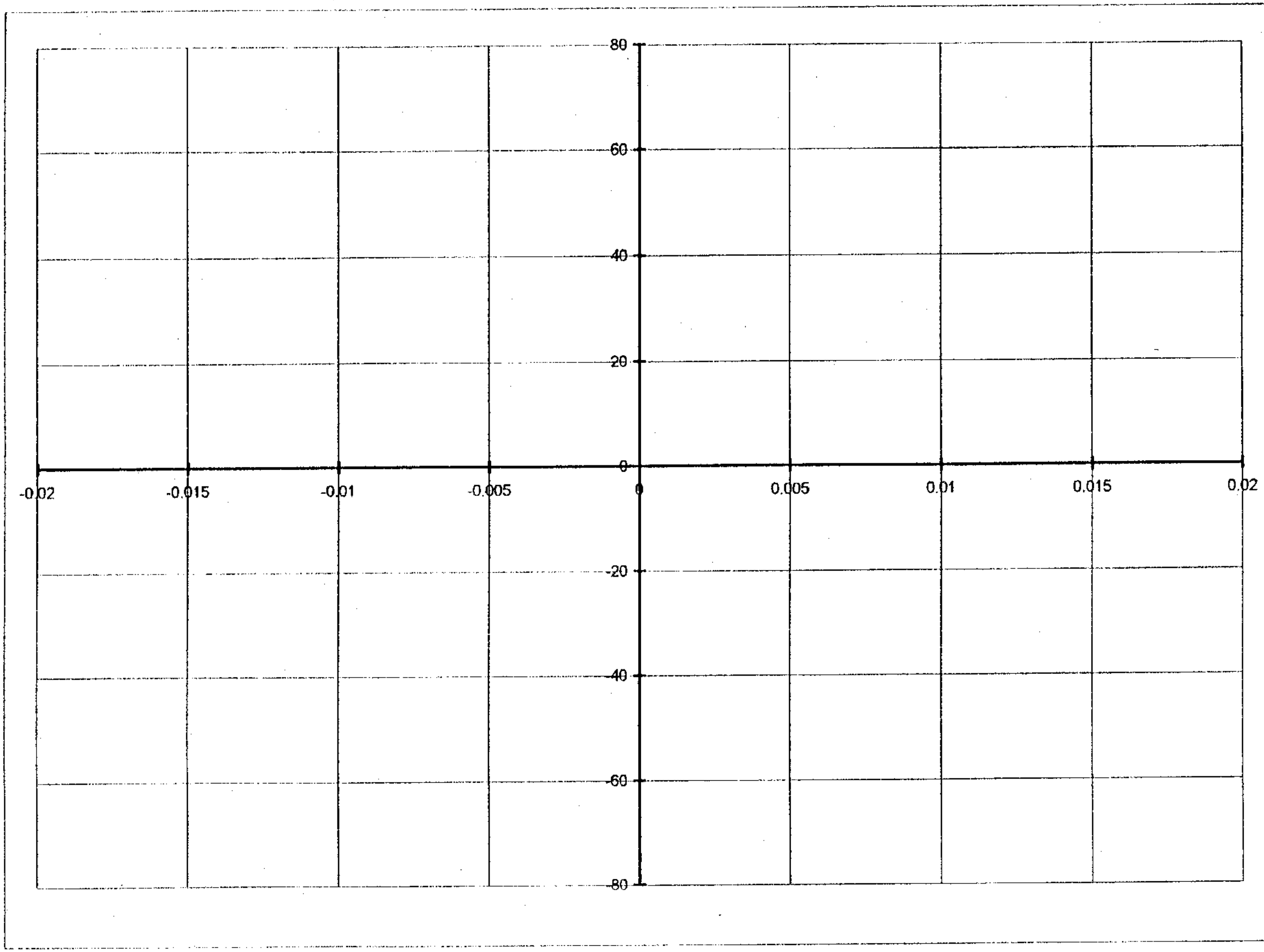


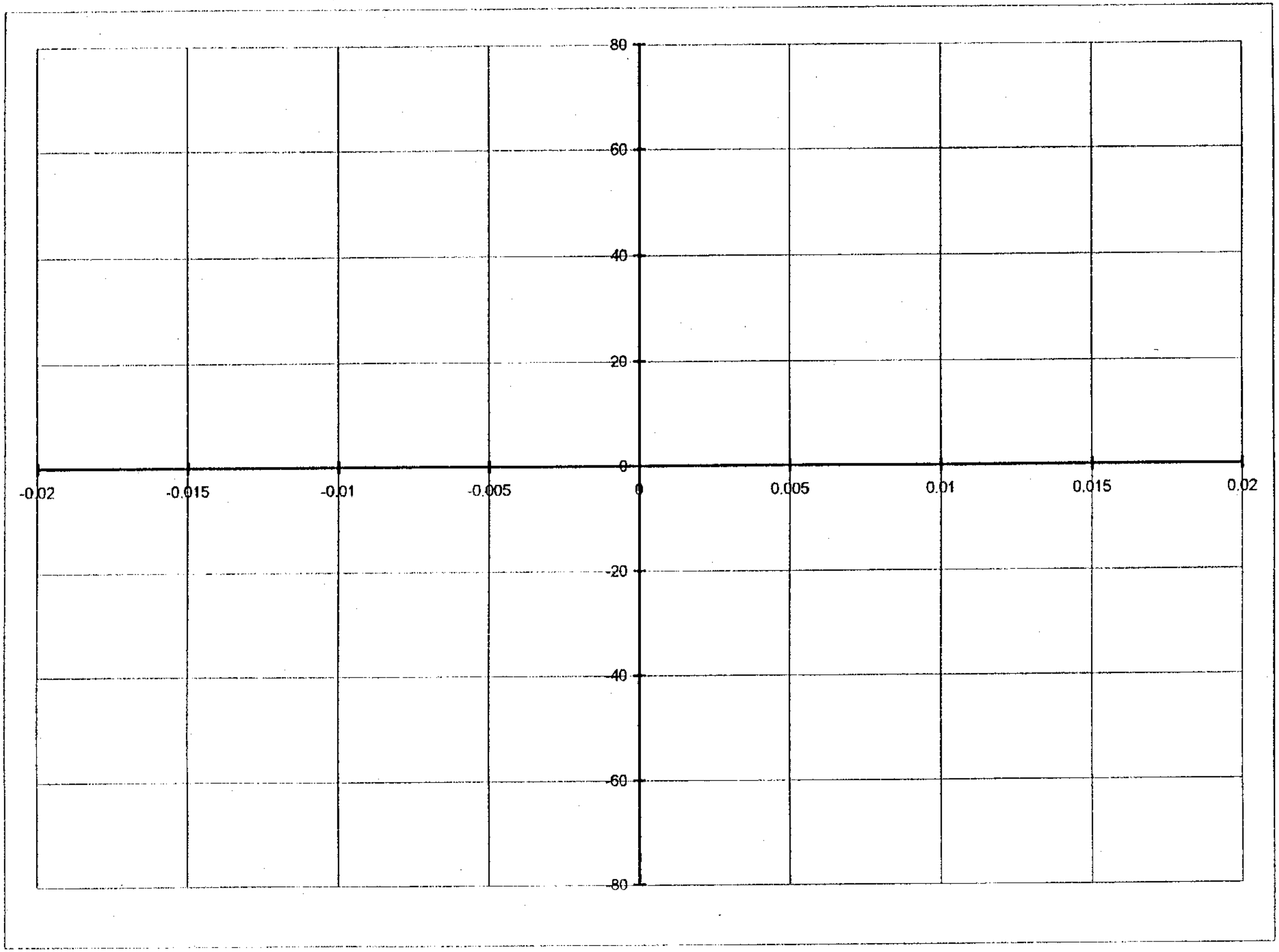






160
140
120
100
80
60
40
20
0
0
0.005
0.01
0.015
0.02
0.025
0.03
0.035
0.04


80
60
40
20
0
-20
-40
-60
-80
-0.02
-0.015
-0.01
-0.005
0
0.005
0.01
0.015
0.02



80
60
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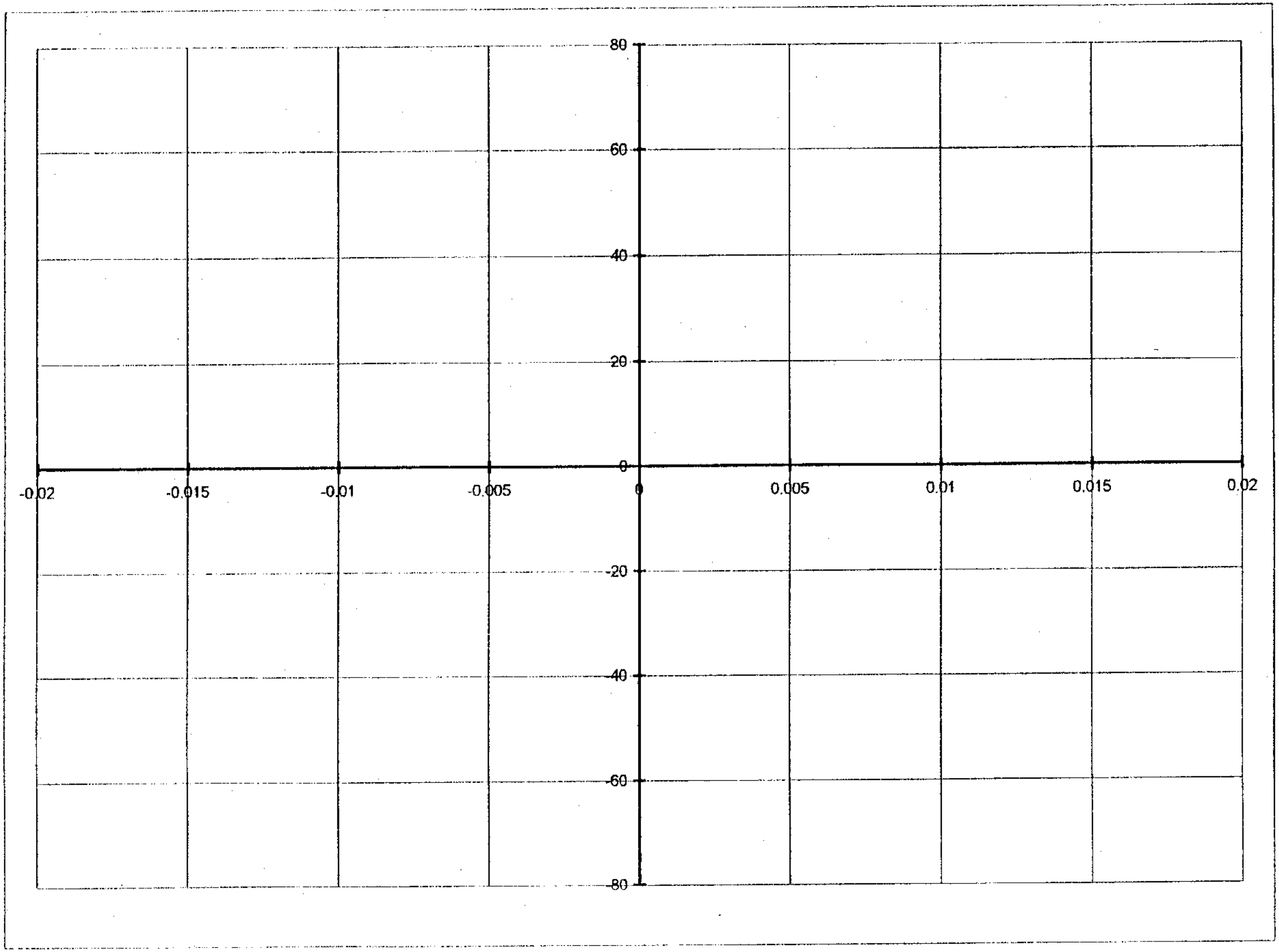
-20
-40
-60
-80
-0.02
-0.015
-0.01
-0.005
0
0.005
0.01
0.015
0.02


80
60
40
20
0
-20
-40
-60
-80
-0.02
-0.015
-0.01
-0.005
0
0.005
0.01
0.015
0.02

Stress history A

1st reversal 0-24 ksi (the Neuber rule)

$$\begin{cases} \dfrac{\left(K_t S_{\max}\right)^2}{E} = \sigma_{\max}\varepsilon_{\max} \\ \varepsilon_{\max} = \dfrac{\sigma_{\max}}{E} + \left(\dfrac{\sigma_{\max}}{K^{'}}\right)^{\frac{1}{n^{'}}} \end{cases} \Rightarrow \begin{cases} \dfrac{\left(2.56\times 24\right)^2}{10600} = \sigma_{\max}\varepsilon_{\max} \\ \varepsilon_{\max} = \dfrac{\sigma_{\max}}{10600} + \left(\dfrac{\sigma_{\max}}{95}\right)^{\frac{1}{0.065}} \end{cases} \Rightarrow \begin{cases} \sigma_{\max} = 59.36\,ksi \\ \varepsilon_{\max} = 0.0060022 \end{cases}$$

2nd reversal 24-0 ksi

$$\Delta S = \left|S_{\max} - S_{\min}\right| = \left|24-0\right| = 24ksi$$

$$\begin{cases} \dfrac{\left(K_t \Delta S\right)^2}{E} = \Delta\sigma\cdot\Delta\varepsilon \\ \dfrac{\Delta\varepsilon}{2} = \dfrac{\Delta\sigma}{2E} + \left(\dfrac{\Delta\sigma}{2K^{'}}\right)^{\frac{1}{n^{'}}} \end{cases} \Rightarrow \begin{cases} \dfrac{\left(2.56\times 24\right)^2}{10600} = \Delta\sigma\cdot\Delta\varepsilon \\ \dfrac{\Delta\varepsilon}{2} = \dfrac{\Delta\sigma}{2\cdot 10600} + \left(\dfrac{\Delta\sigma}{2\cdot 95}\right)^{\frac{1}{0.065}} \end{cases} \Rightarrow \begin{cases} \Delta\sigma = 61.45\,ksi \\ \Delta\varepsilon = 0.0057979 \end{cases}$$

Minimum stress and strain after the 2nd reversal

$$\begin{cases}\sigma_{\min} = \sigma_{\max} - \Delta\sigma \\ \varepsilon_{\min} = \varepsilon_{\max} - \Delta\varepsilon\end{cases} \Rightarrow \begin{cases}\sigma_{\min} = 59.36 - 61.45 \\ \varepsilon_{\min} = 0.006022 - 0.0057979\end{cases} \Rightarrow \begin{cases}\sigma_{\min} = -2.09\,ksi \\ \varepsilon_{\min} = 0.0002043\end{cases}$$

3nd reversal 0-24 ksi

$$\Delta S = \left|S_{\max} - S_{\min}\right| = \left|0 - 24\right| = 24ksi$$

$$\begin{cases}\dfrac{\left(K_t \Delta S\right)^2}{E} = \Delta\sigma \cdot \Delta\varepsilon \\ \dfrac{\Delta\varepsilon}{2} = \dfrac{\Delta\sigma}{2E} + \left(\dfrac{\Delta\sigma}{2K'}\right)^{\frac{1}{n'}}\end{cases} \Rightarrow \begin{cases}\dfrac{\left(2.56 \times 24\right)^2}{10600} = \Delta\sigma \cdot \Delta\varepsilon \\ \dfrac{\Delta\varepsilon}{2} = \dfrac{\Delta\sigma}{2 \cdot 10600} + \left(\dfrac{\Delta\sigma}{2 \cdot 95}\right)^{\frac{1}{0.065}}\end{cases} \Rightarrow \begin{cases}\Delta\sigma = 61.45\,ksi \\ \Delta\varepsilon = 0.0057979\end{cases}$$

Maximum stress and strain after the 3rd reversal

$$\begin{cases}\sigma_{\max} = \sigma_{\min} + \Delta\sigma \\ \varepsilon_{\max} = \varepsilon_{\min} + \Delta\varepsilon\end{cases} \Rightarrow \begin{cases}\sigma_{\max} = -2.09 + 61.45 \\ \varepsilon_{\max} = 0.0002043 + 0.0057979\end{cases} \Rightarrow \begin{cases}\sigma_{\max} = 59.36\,ksi \\ \varepsilon_{\max} = 0.006022\end{cases}$$

The Maximum stress and strain after the 1st and the 3rd reversal are the same. The stress –strain loop has been closed, i.e. the stress strain cycle has been found!

$$\begin{cases} \sigma_{\max} = 59.36\,ksi \\ \varepsilon_{\max} = 0.006022 \end{cases} \quad \begin{cases} \sigma_{\min} = -2.09\,ksi \\ \varepsilon_{\min} = 0.0002043 \end{cases} \quad \begin{cases} \Delta\sigma = 61.45\,ksi \\ \Delta\varepsilon = 0.0057979 \end{cases}$$

$$\sigma_{mean} = \frac{\sigma_{\max} + \sigma_{\min}}{2} = \frac{59.36 - 2.09}{2} = 28.635\,ksi$$

The Manson-Coffin curve corrected for the mean stress effect (Morrow’s correction)

$$\frac{\Delta\varepsilon}{2} = \frac{\sigma_f' - \sigma_{mean}}{E}\left(2N_f\right)^b + \varepsilon_f'\left(2N_f\right)^c$$

$$\frac{0.0057979}{2} = \frac{160 - 28.635}{10600}\left(2N_f\right)^{-0.124} + 0.22\left(2N_f\right)^{-0.59}$$

$$\Downarrow$$

$N_f = 97880$ *cycles;* Calculated

$N_f = 110000$ *cycles;* Experimental by NASA