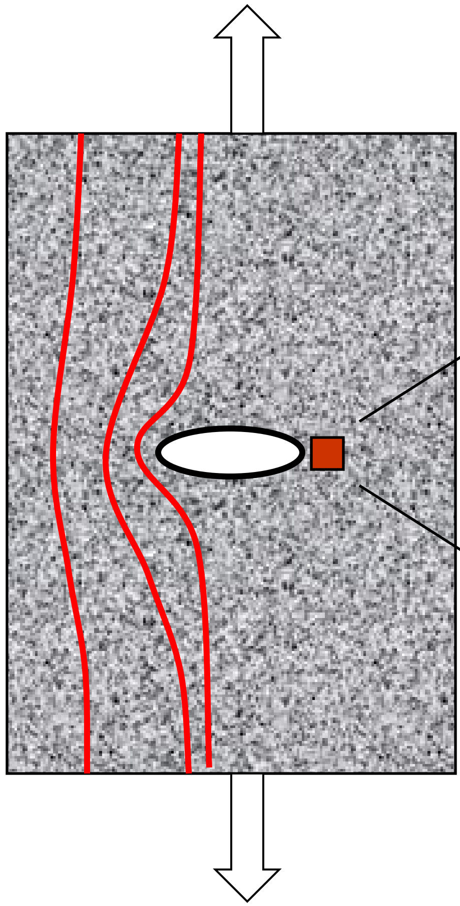
Stress Concentration

Professor Darrell F. Socie

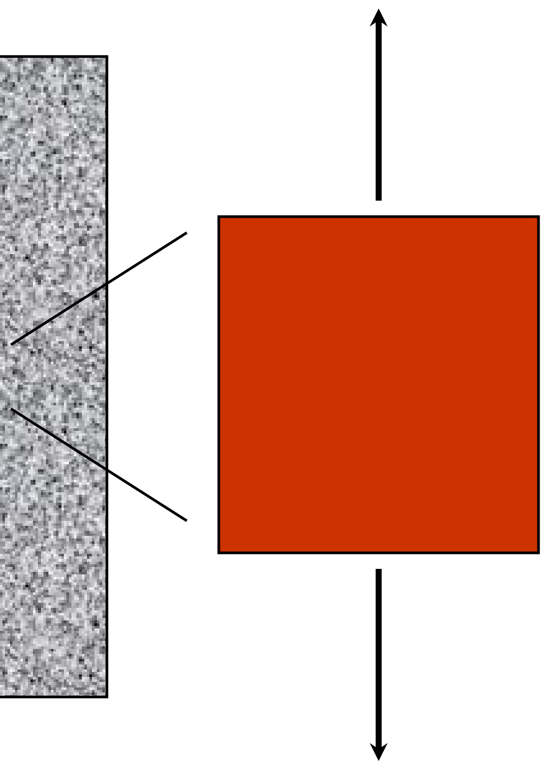
Outline

1. **Stress Concentration**
2. Notch Rules
3. Fatigue Notch Factor
4. Stress Intensity Factors for Notches
5. Frost Data and K_f
6. Small Crack Growth
7. Small Notches

Stress Concentration Factor



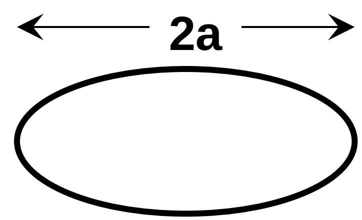
Applied stress



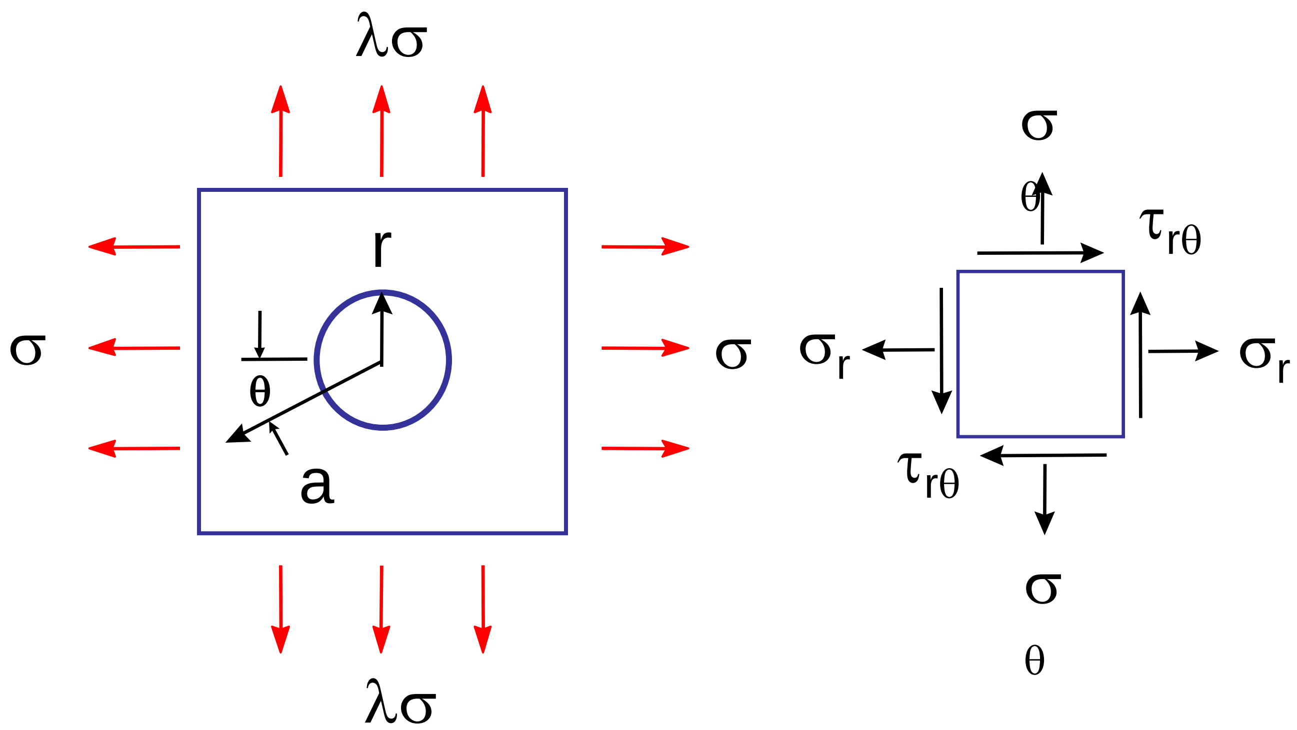
Local stress

$$\sigma_{local} = \sigma_{applied}\left(1 + 2\sqrt{\frac{a}{\rho}}\right)$$

Inglis Solution 1910



Circular Notch



Stresses

$$\frac{\sigma_r}{\sigma} = \frac{1+\lambda}{2}\left(1-\left(\frac{r}{a}\right)^2\right) + \frac{1-\lambda}{2}\left(1+3\left(\frac{r}{a}\right)^4 - 4\left(\frac{r}{a}\right)^2\right)\cos 2\theta$$

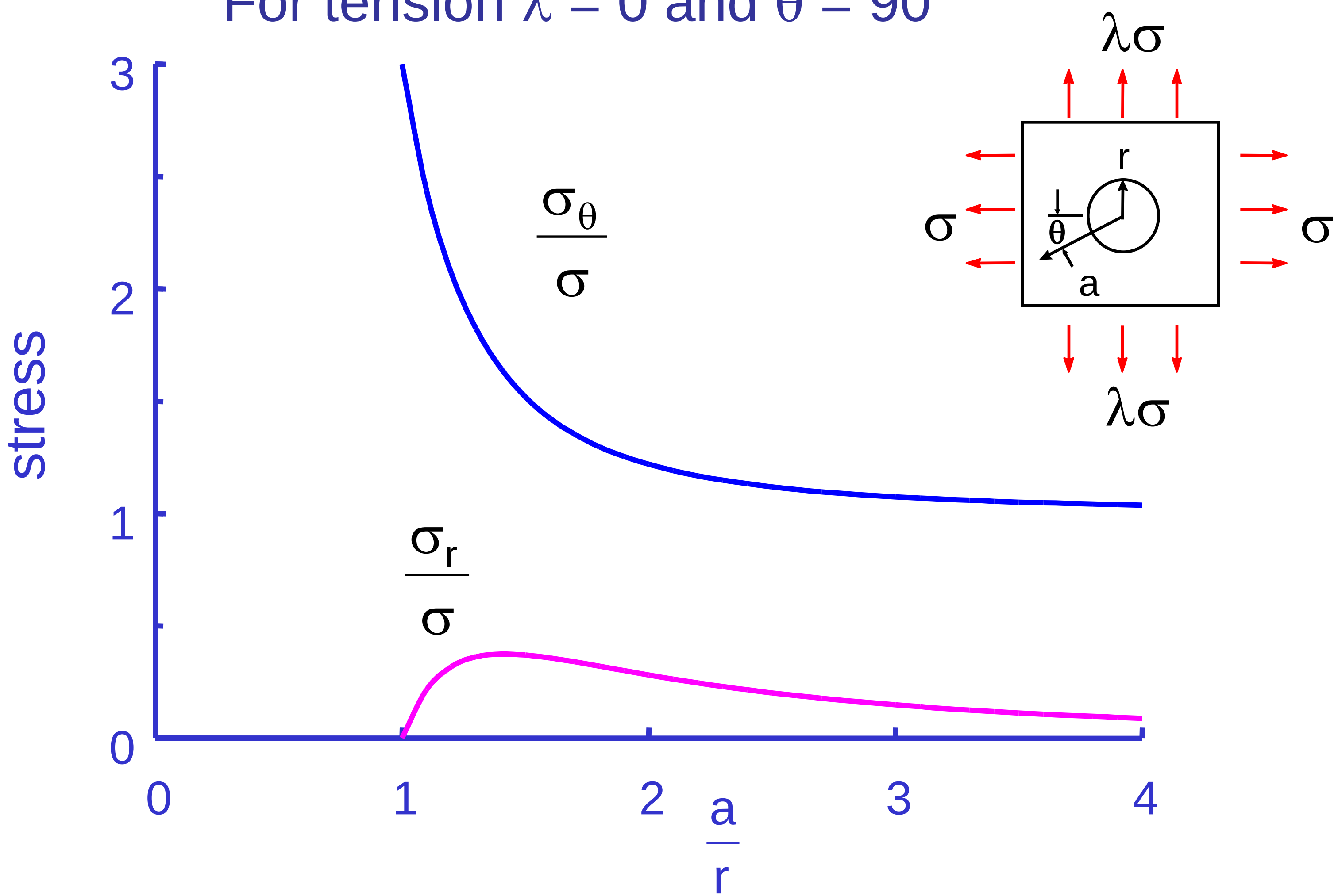
$$\frac{\sigma_\theta}{\sigma} = \frac{1+\lambda}{2}\left(1+\left(\frac{r}{a}\right)^2\right) - \frac{1-\lambda}{2}\left(1+3\left(\frac{r}{a}\right)^4\right)\cos 2\theta$$

$$\frac{\tau_{r\theta}}{\sigma} = -\frac{1-\lambda}{2}\left(1-3\left(\frac{r}{a}\right)^4 + 2\left(\frac{r}{a}\right)^2\right)\sin 2\theta$$

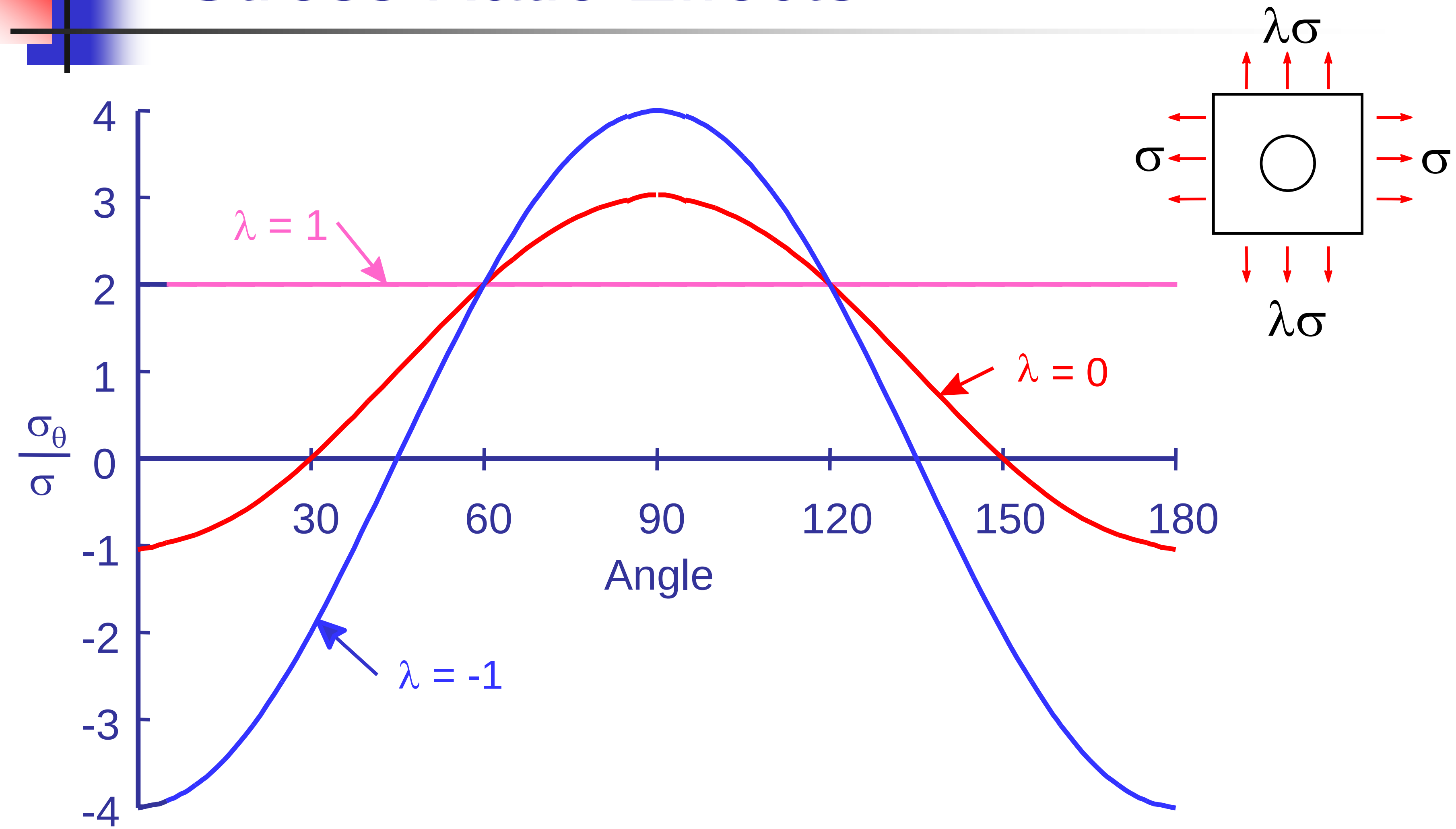
Independent of size, dependant only on r/a

Stress Distribution

For tension $\lambda = 0$ and $\theta = 90$

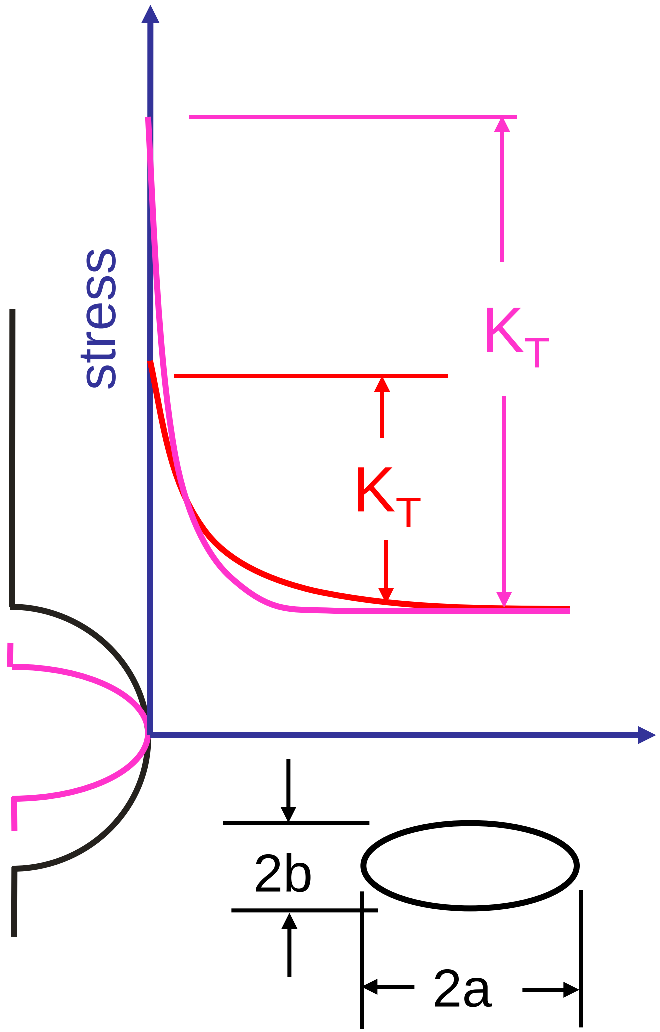


Stress Ratio Effects



stresses around the circumference of a hole

Elliptical Notches

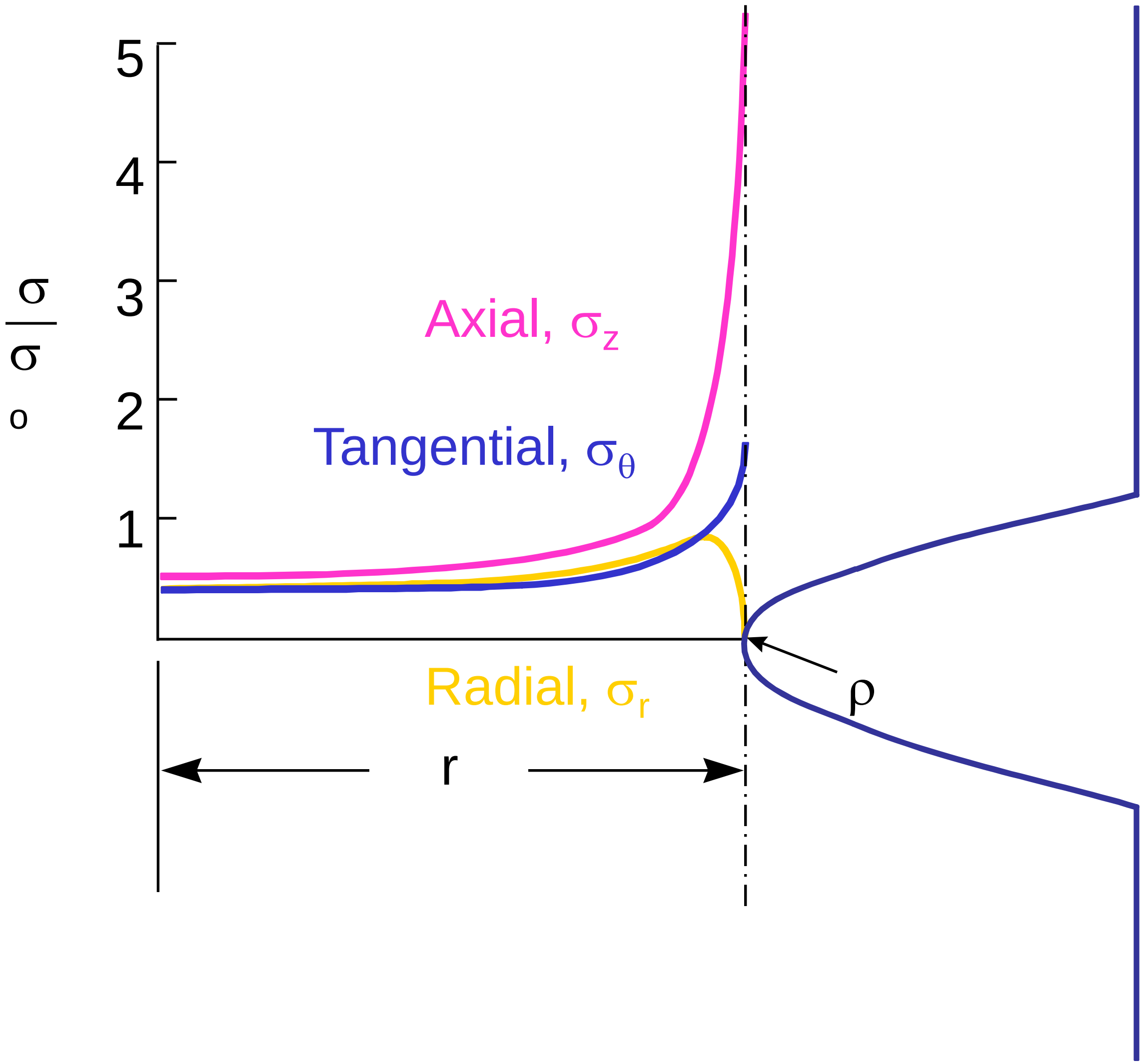


$$K_T = 1 + 2\sqrt{\frac{a}{\rho}} \qquad \rho = \frac{b^2}{a}$$

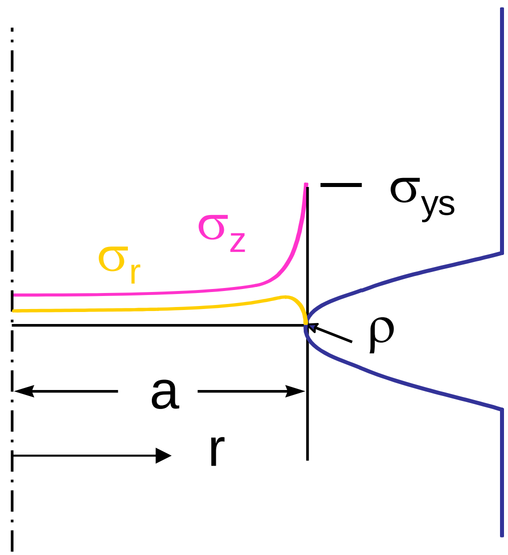
Sharp Notch:
- high K_T
- high gradient

Blunt Notch:
- low K_T
- low gradient

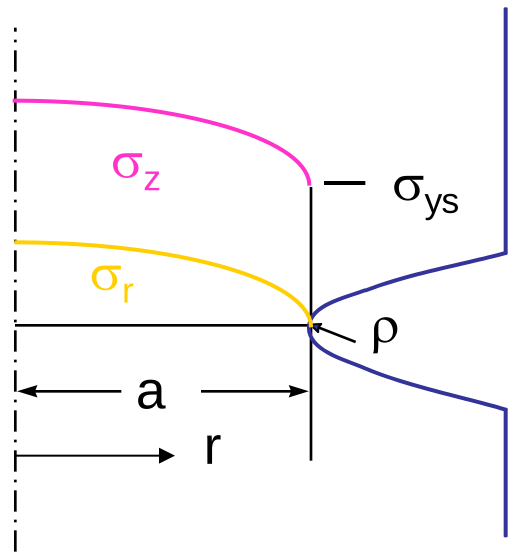
Stress Concentration in a Bar



Bridgeman Analysis (1943)



Elastic stress distribution



Plastic stress distribution

$$\tau = \frac{\sigma_z - \sigma_r}{2} = \text{constant}$$

Stresses

$$\sigma_z = \sigma_o \left[1 + \ln\left(\frac{a^2 + 2a\rho - r^2}{2a\rho} \right) \right]$$

$$P_z = \int_0^a 2\pi r \, \sigma_z \, dr$$

$$P_{max} = \pi a^2 \sigma_{flow} \left(1 + \frac{2\rho}{a} \right) \ln\left(1 + \frac{a}{2\rho} \right)$$

$$P_{max} = A_{net} \, \sigma_{flow} \, CF$$

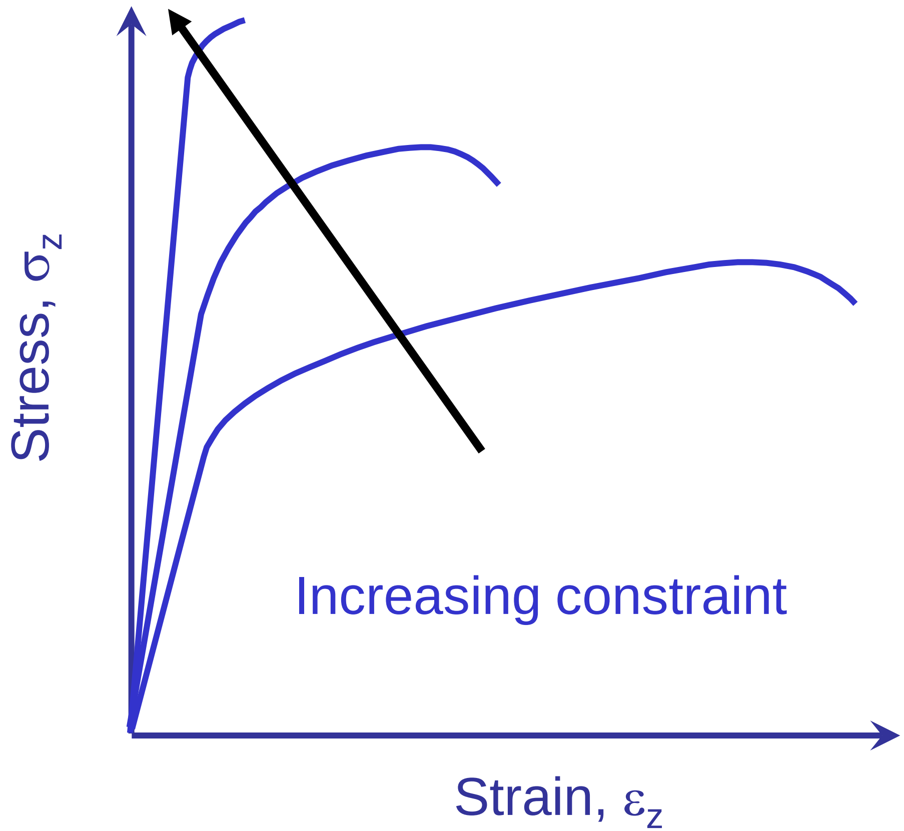
CF constraint factor

Constraint Factors

a/ρ	CF
0	1
1	1.21
2	1.38
4	1.64
8	1.73
20	2.63
∞	2.96

$$P_{max} = A_{net}\, \sigma_{flow}\, CF$$

Effect of Constraint



Higher strength and lower ductility

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Notch Rules

Neuber

$$K_t^2 S e = \sigma \varepsilon = \frac{\sigma^2}{E} + \sigma \left(\frac{\sigma}{K} \right)^{\frac{1}{n}}$$

Glinka

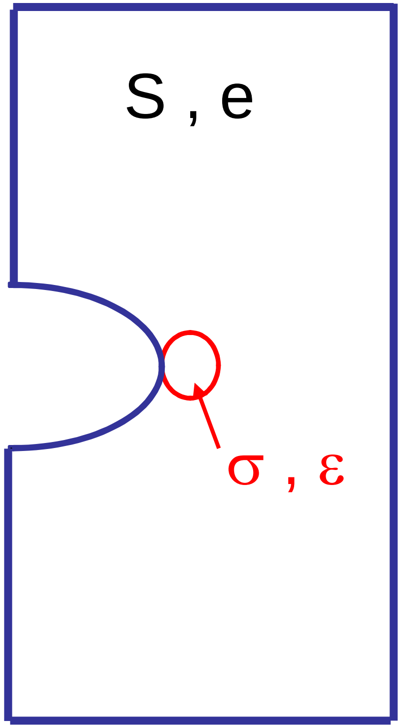
$$K_t^2 S e = \int \sigma d\varepsilon = \frac{\sigma^2}{E} + \frac{1}{1+n} \sigma \left(\frac{\sigma}{K} \right)^{\frac{1}{n}}$$

Seeger

$$K_p^2 S^* e^* = \sigma \varepsilon = \frac{\sigma^2}{E} + \sigma \left(\frac{\sigma}{K} \right)^{\frac{1}{n}}$$

$$K_p = \frac{S_{Limit} K_t}{\sigma_y} \qquad S^* = \frac{K_t}{K_p} S$$

Define K_σ and K_ε after Yielding

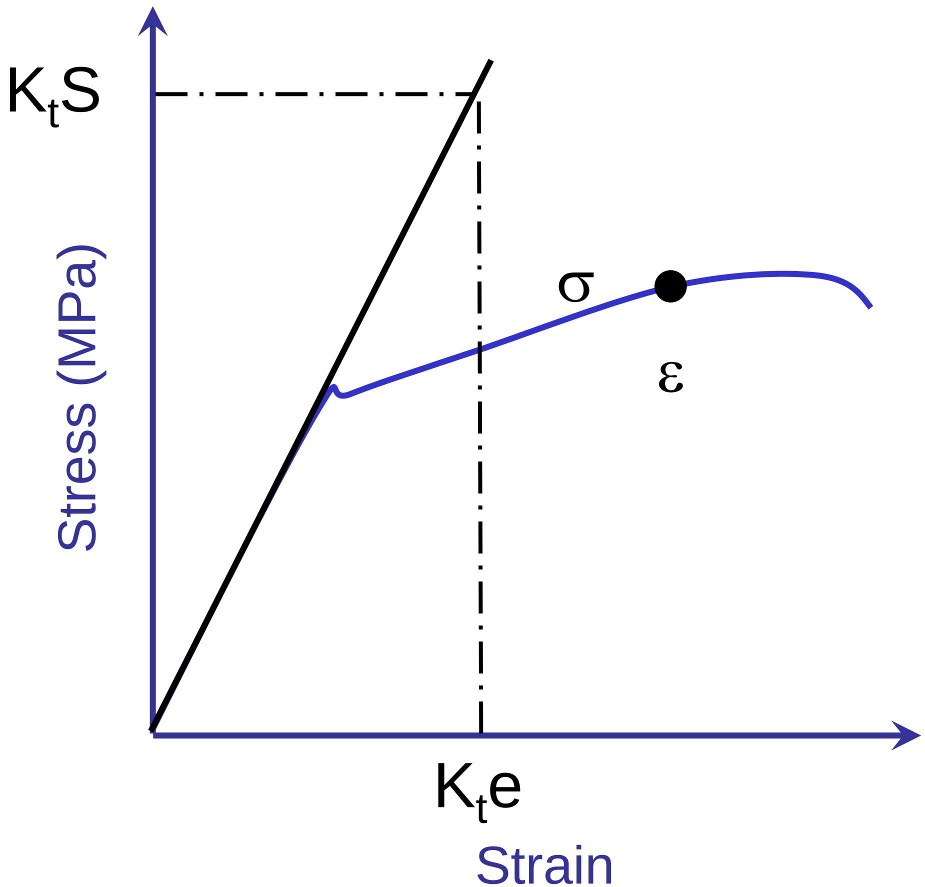


Define: nominal stress, S
nominal strain, e
notch stress, σ
notch strain, ε

Stress concentration $$K_\sigma = \frac{\sigma}{S}$$

Strain concentration $$K_\varepsilon = \frac{\varepsilon}{e}$$

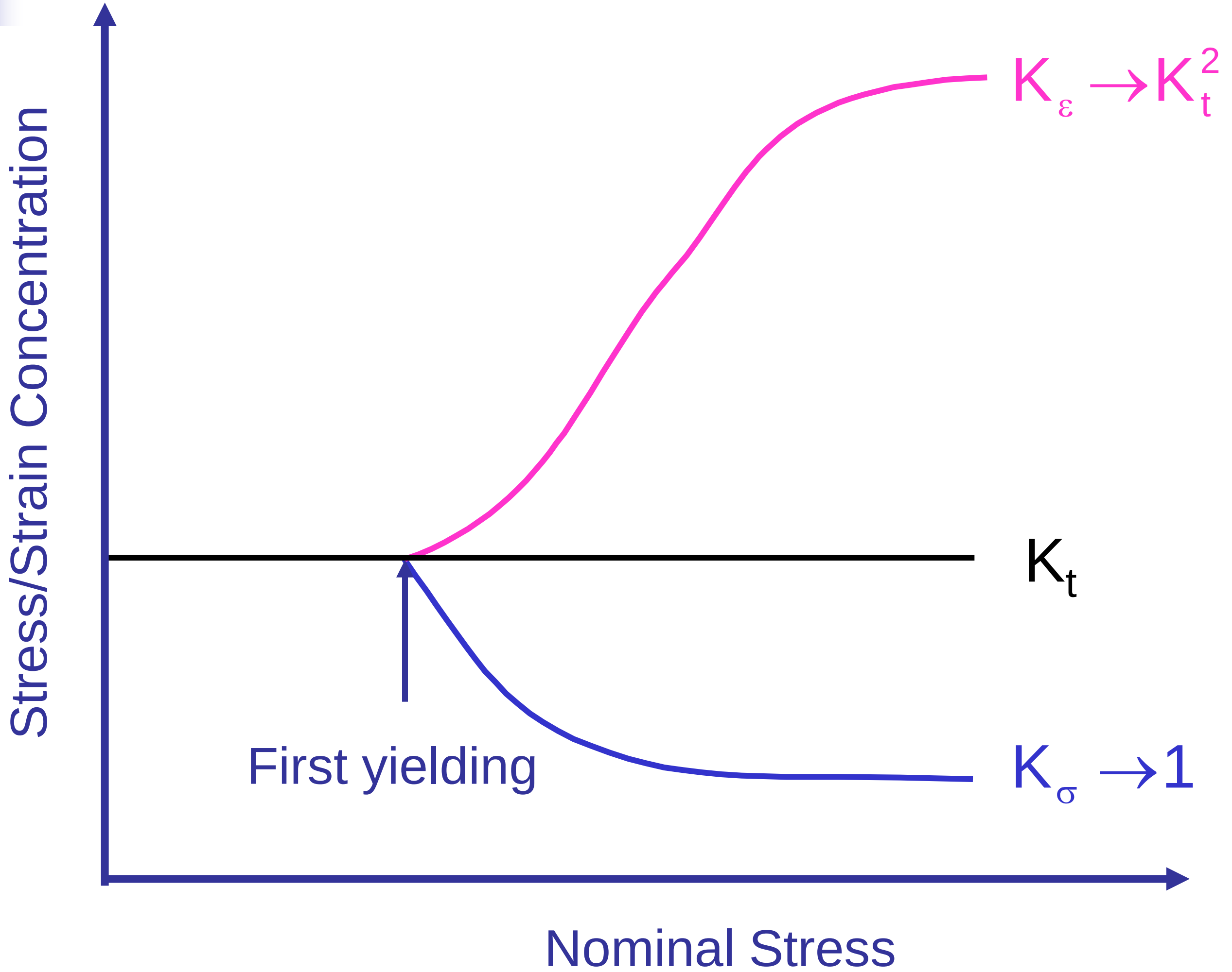
K_σ and K_ε



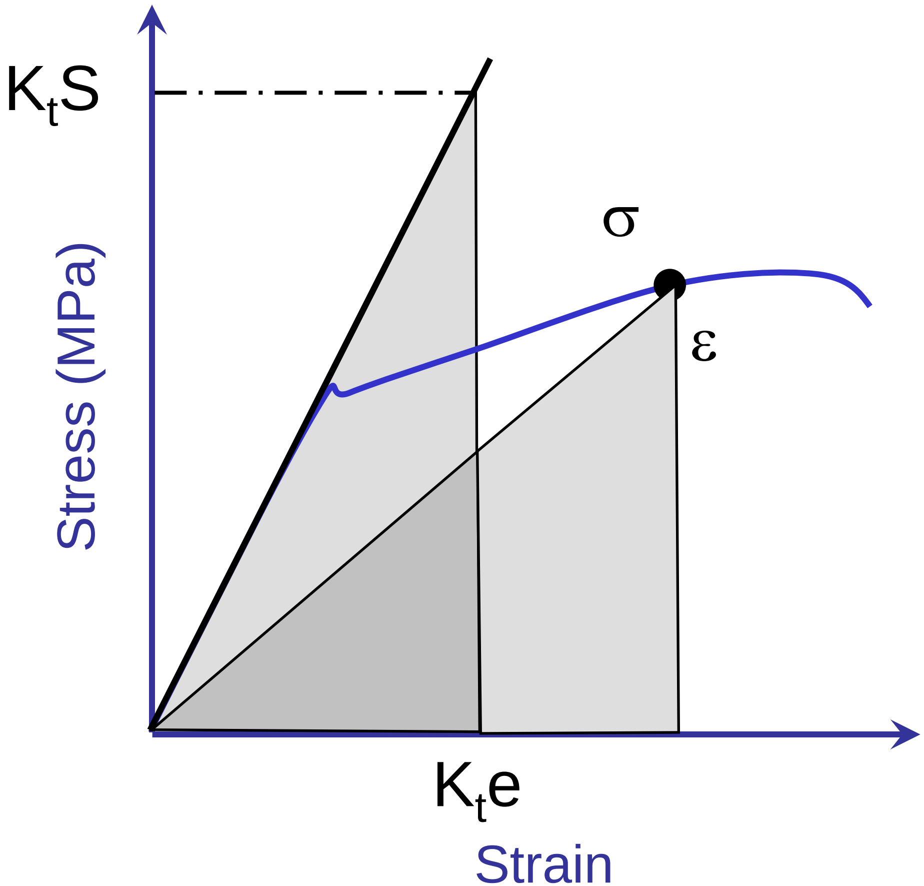
$$K_\sigma = \frac{\sigma}{S}$$

$$K_\varepsilon = \frac{\varepsilon}{e}$$

Stress and Strain Concentration



Neuber's Rule



Actual stress

$$\underbrace{K_t S}\, K_t e = \sigma \varepsilon$$

Stress calculated with elastic assumptions

Neuber's Rule for Fatigue

Stress and strain amplitudes

$$\frac{K_t \Delta S}{2} \frac{K_t \Delta e}{2} = \frac{\Delta\sigma}{2} \frac{\Delta\varepsilon}{2}$$

Elastic nominal stress

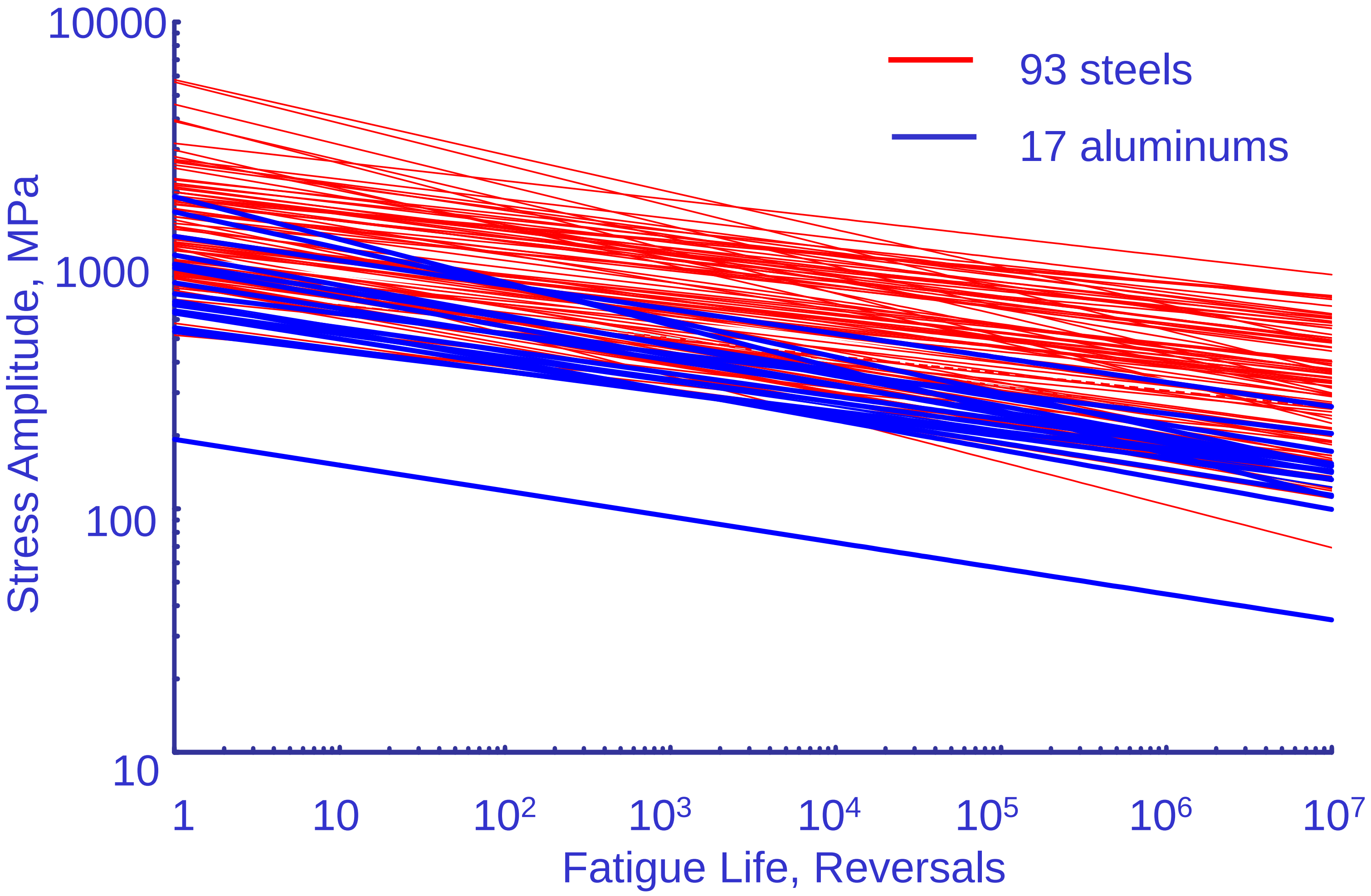
$$\frac{\Delta e}{2} = \frac{\Delta S}{2E}$$

Substitute and rearrange

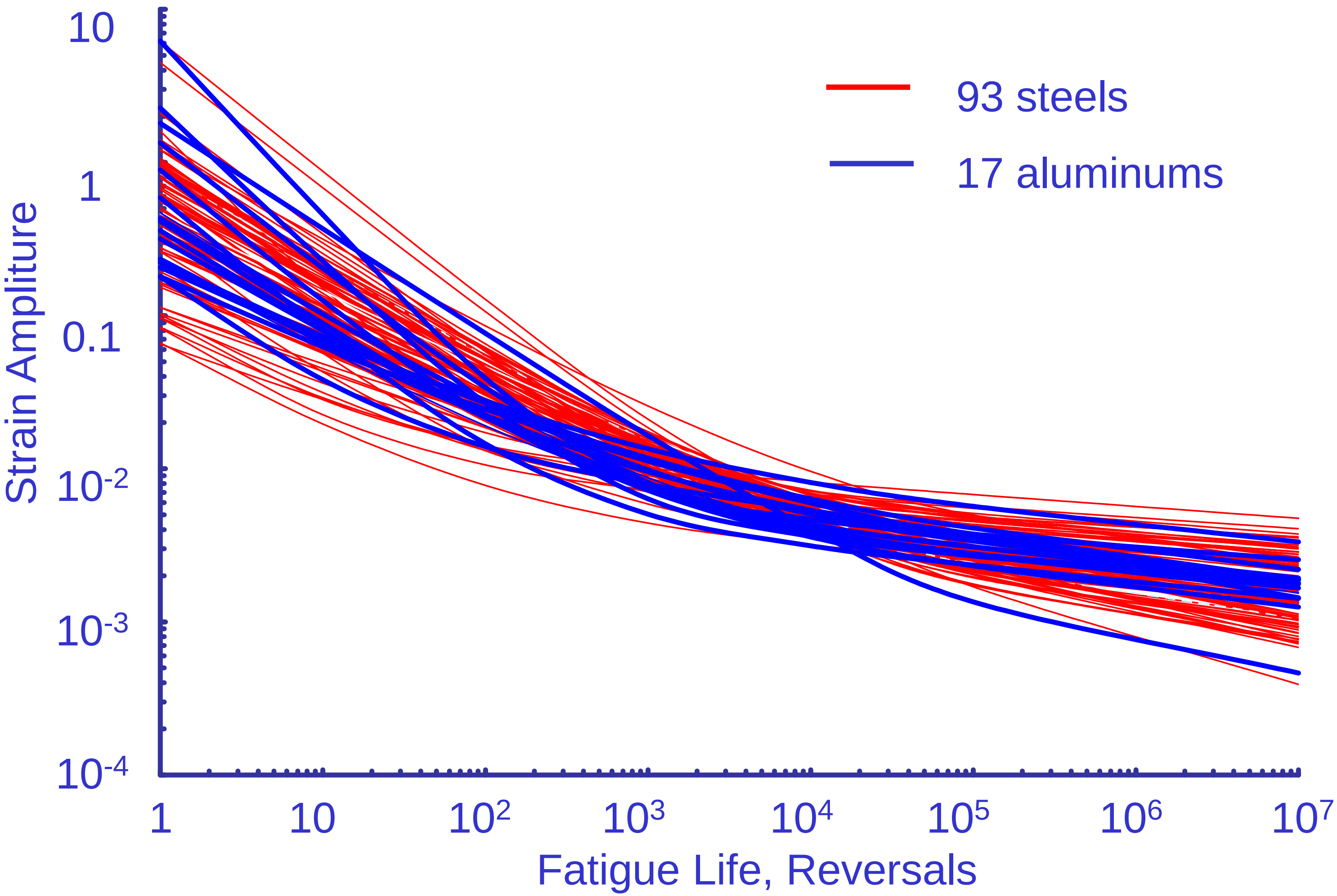
$$K_t \frac{\Delta S}{2} = \sqrt{E \frac{\Delta\sigma}{2} \frac{\Delta\varepsilon}{2}}$$

The product of stress times strain controls fatigue life

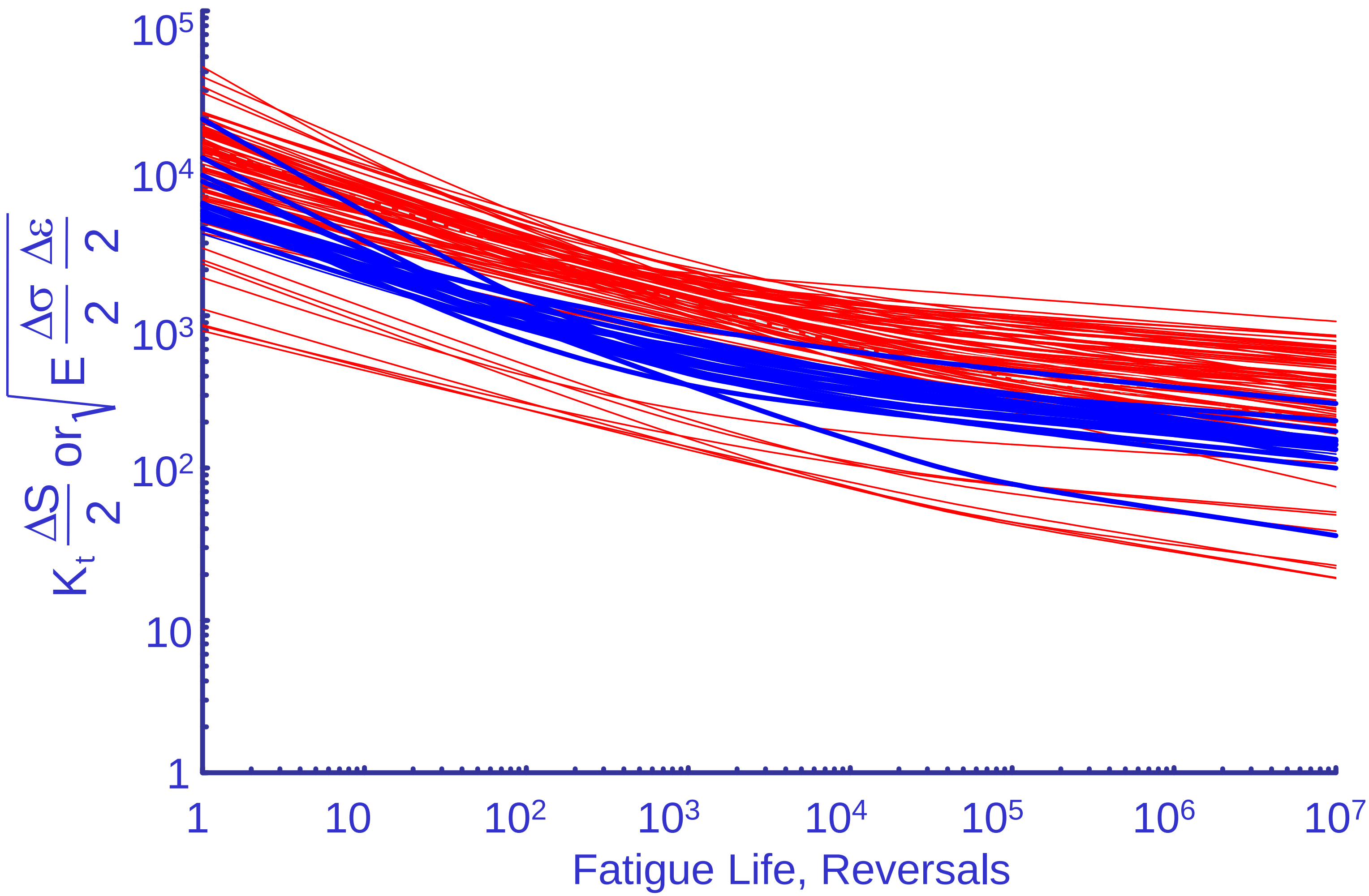
SN Materials Data



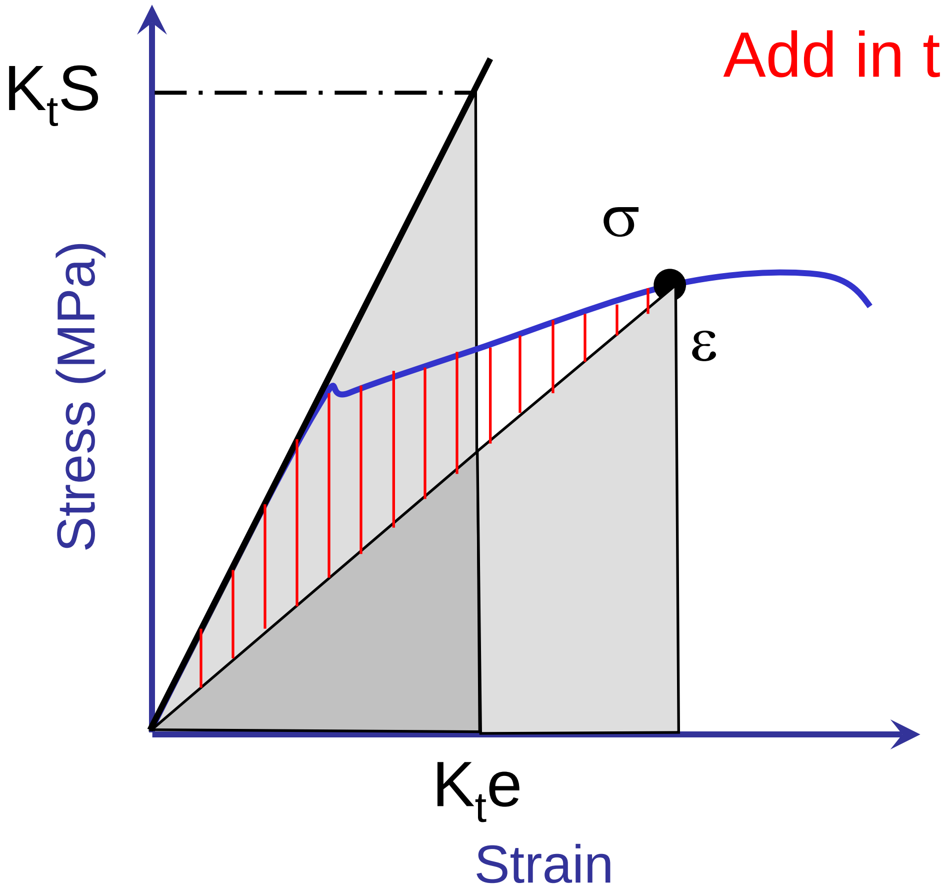
εN Materials Data



$$\sqrt{E\,\frac{\Delta\sigma}{2}\,\frac{\Delta\varepsilon}{2}}$$



Glinka's Rule



Add in the "missing" strain energy

$$K_t S K_t e = 2\int \sigma \, d\varepsilon$$

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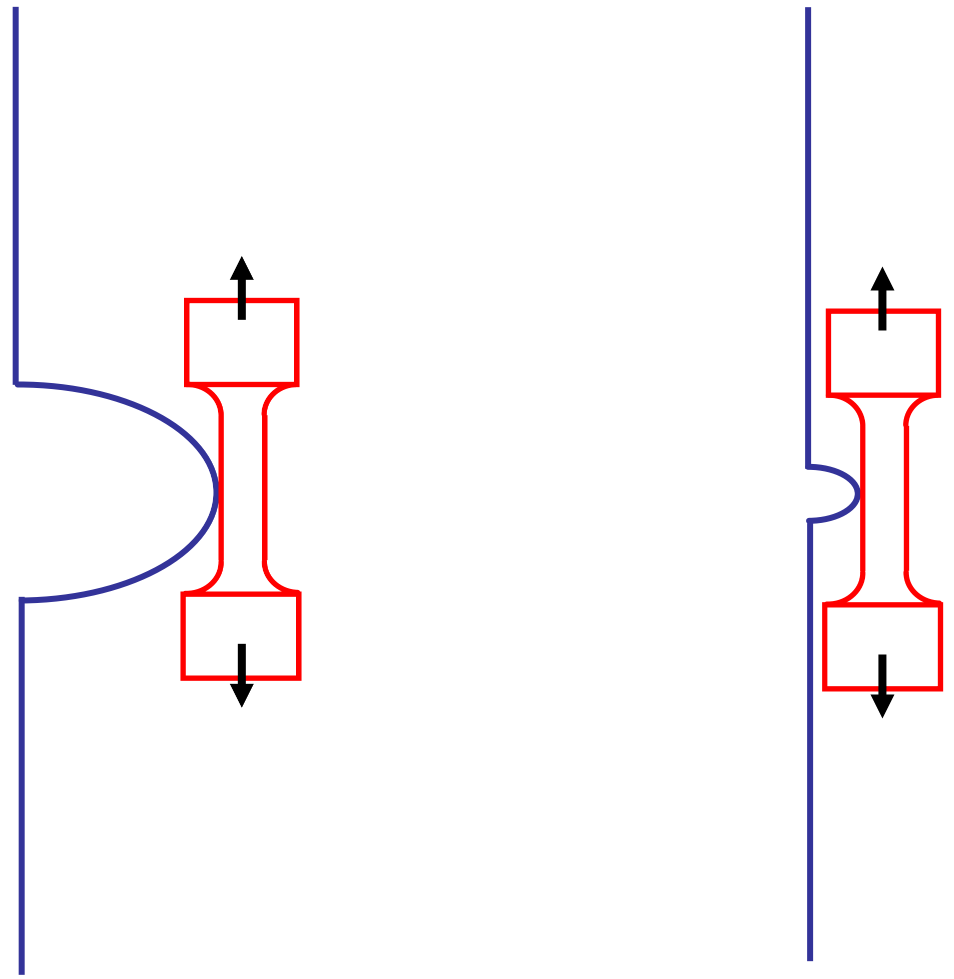
A Dilemma

Stress analysis and stress concentration factors are independent of size and are related only to the ratio of the geometric dimensions to the loads

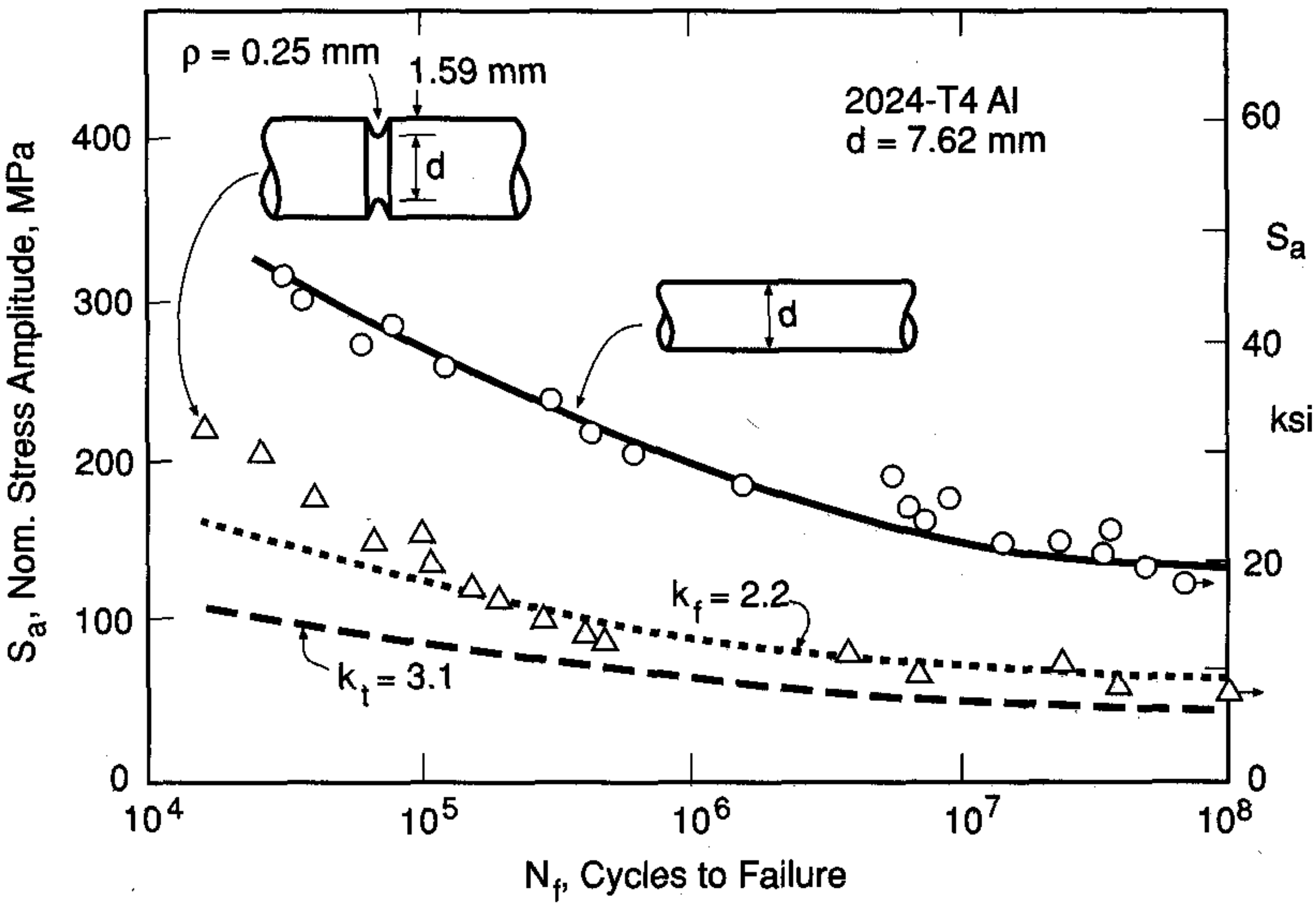
Fatigue is a size dependant phenomena

How do you put the two together ?

Similitude

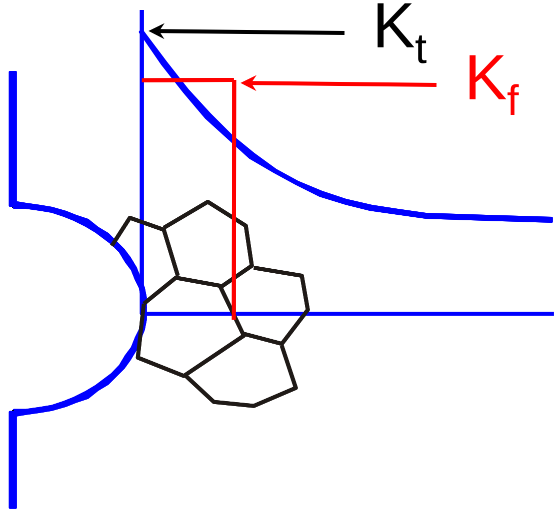


Fatigue of Notches

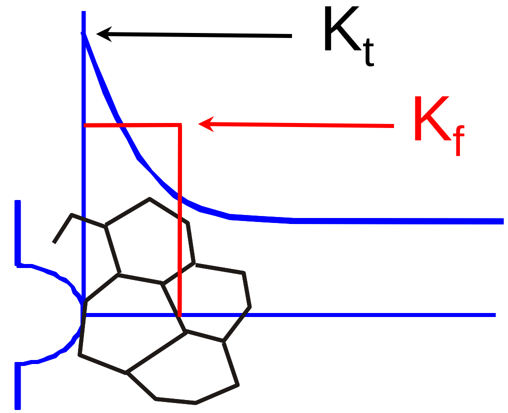


From Dowling, Mechanical Behavior of Materials, 1999

Notch Size

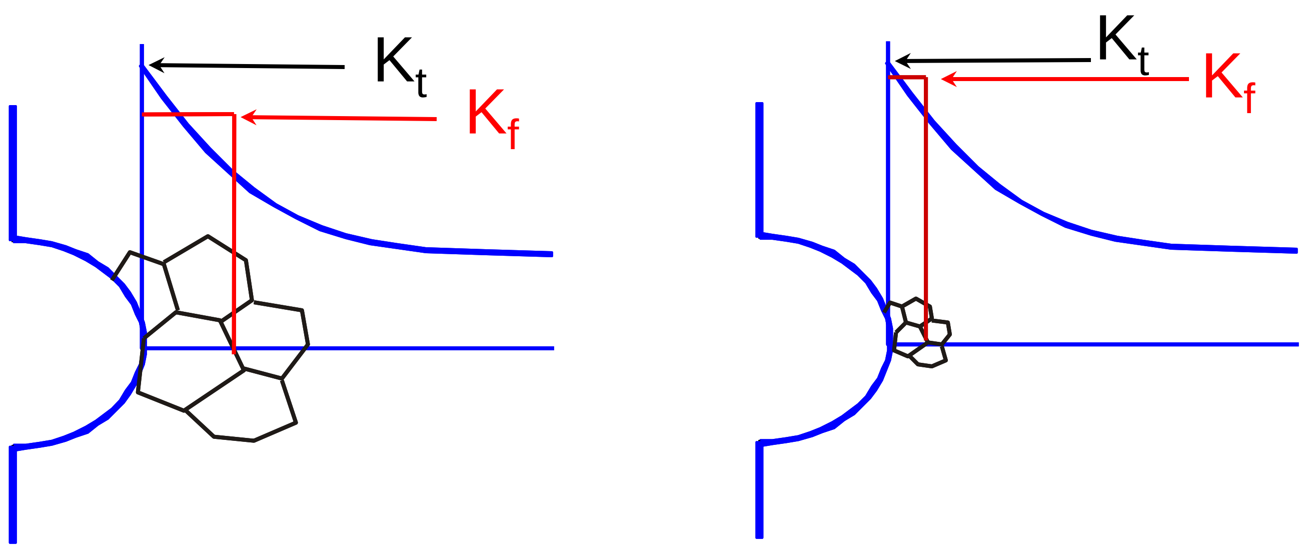


Large Notch



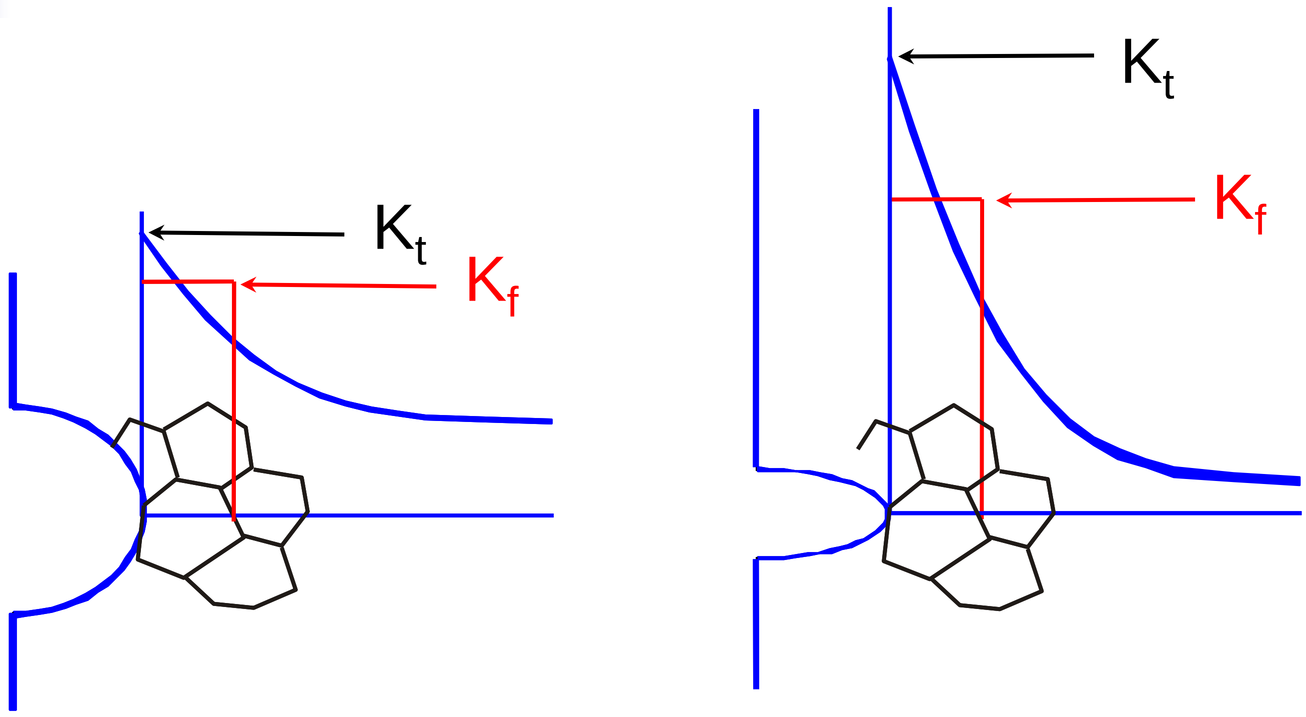
Small Notch

Microstructure Size



Low Strength High Strength

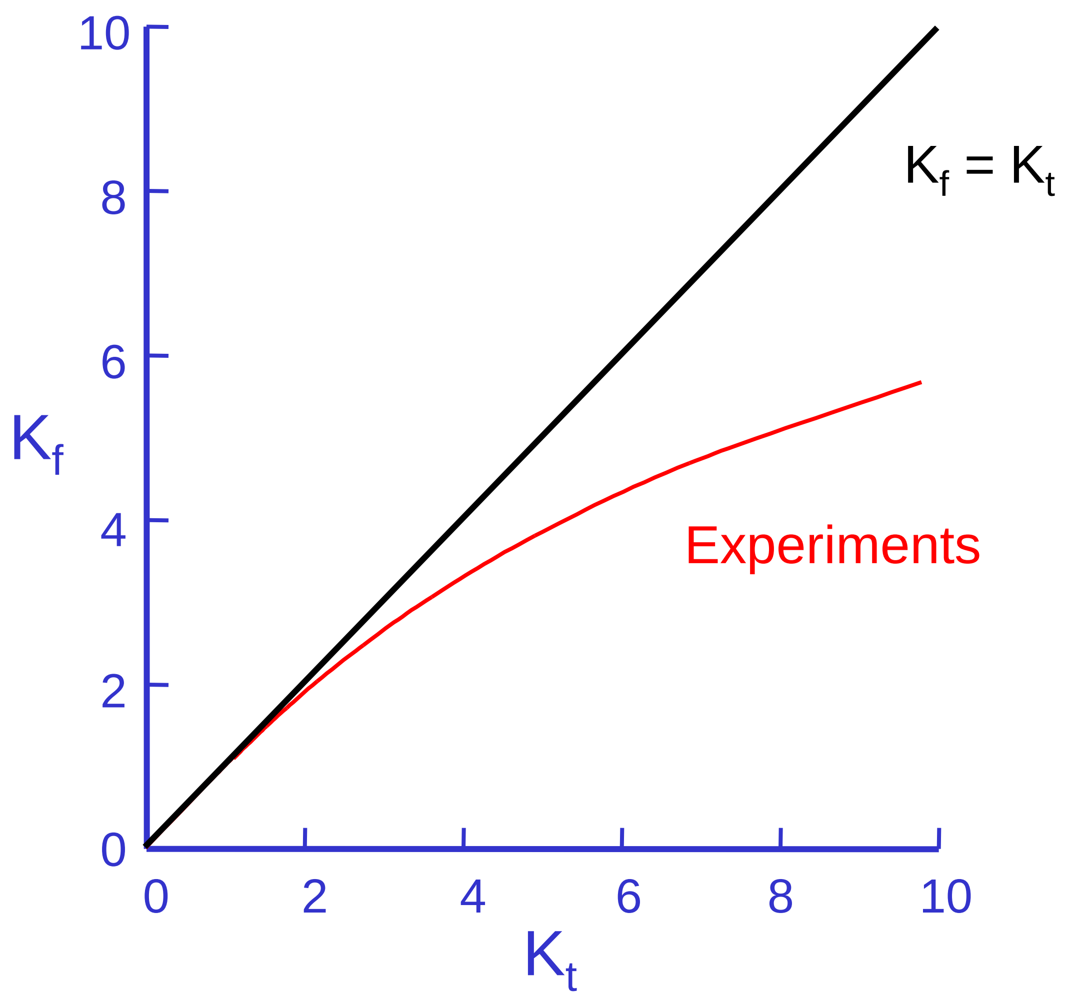
Stress Gradient



Low K_t

High K_t

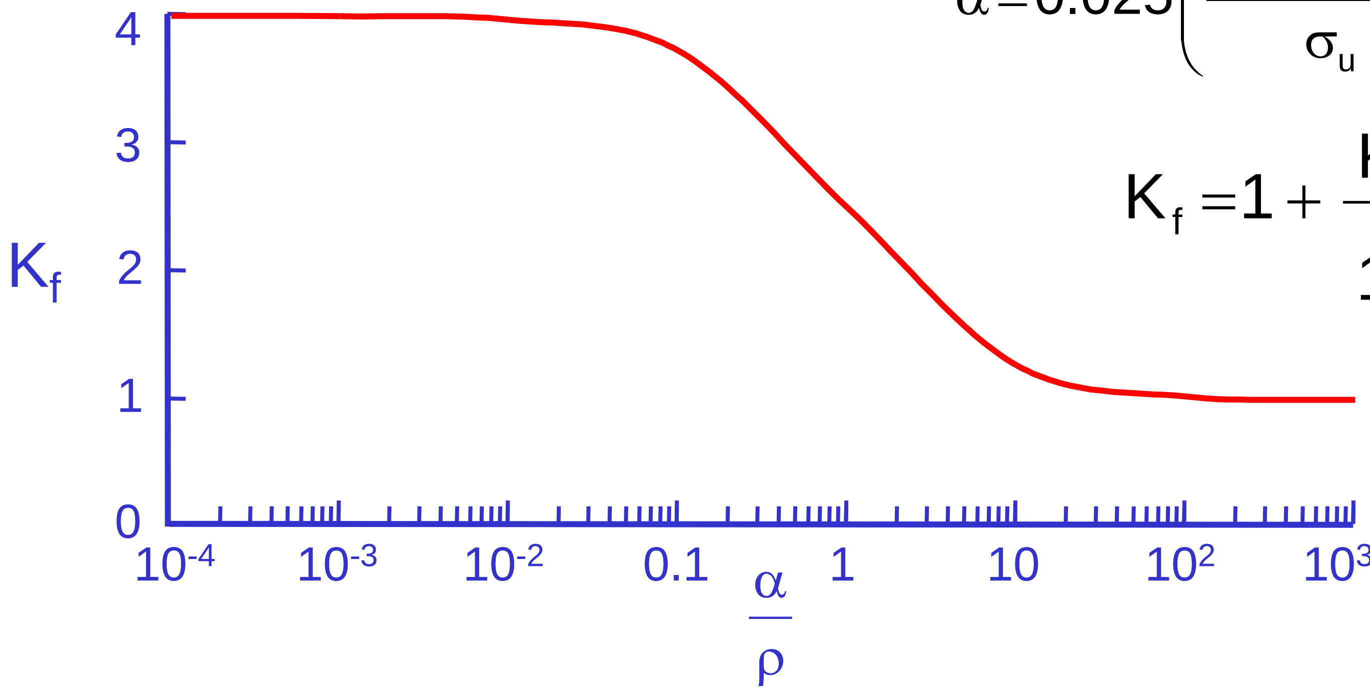
K_t vs K_f



Peterson's Equation

$$\alpha = 0.025\left(\frac{2070\text{MPa}}{\sigma_u}\right)^{1.8} \text{mm}$$

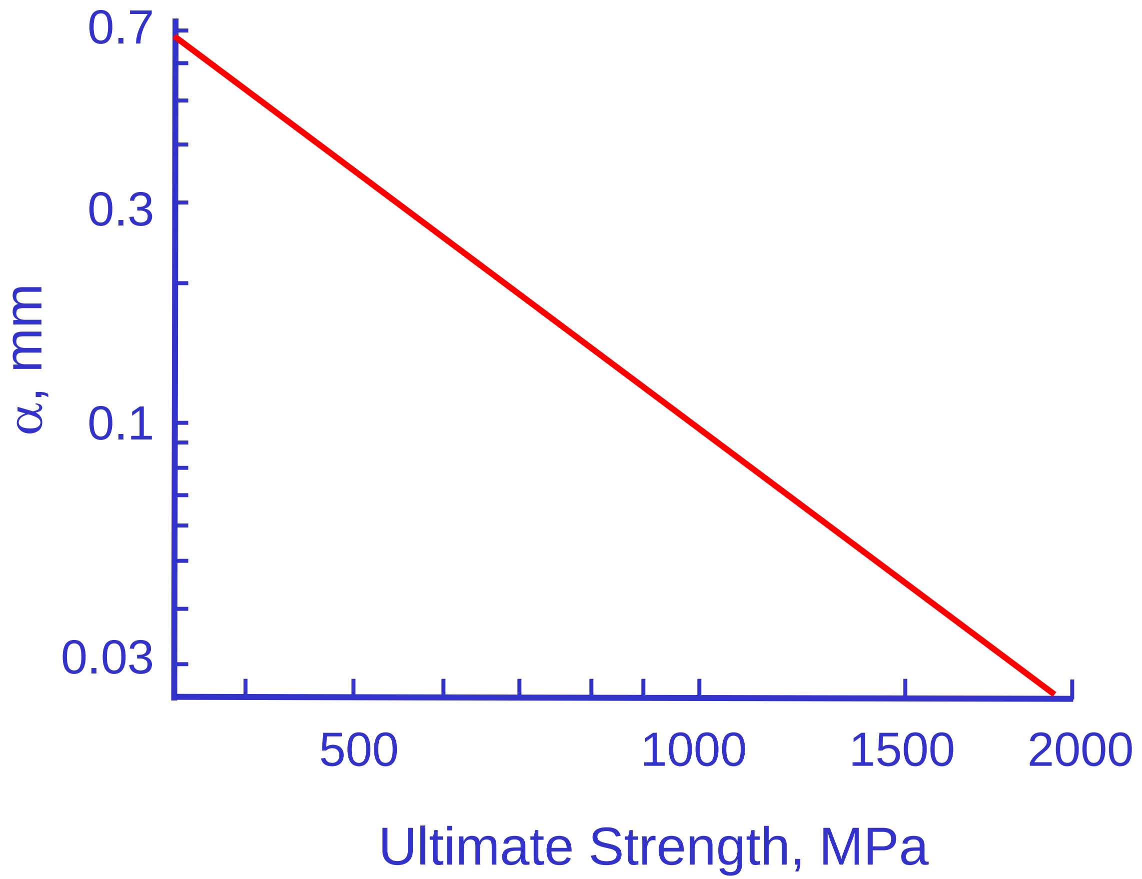
$$K_f = 1 + \frac{K_t - 1}{1 + \frac{\alpha}{\rho}}$$



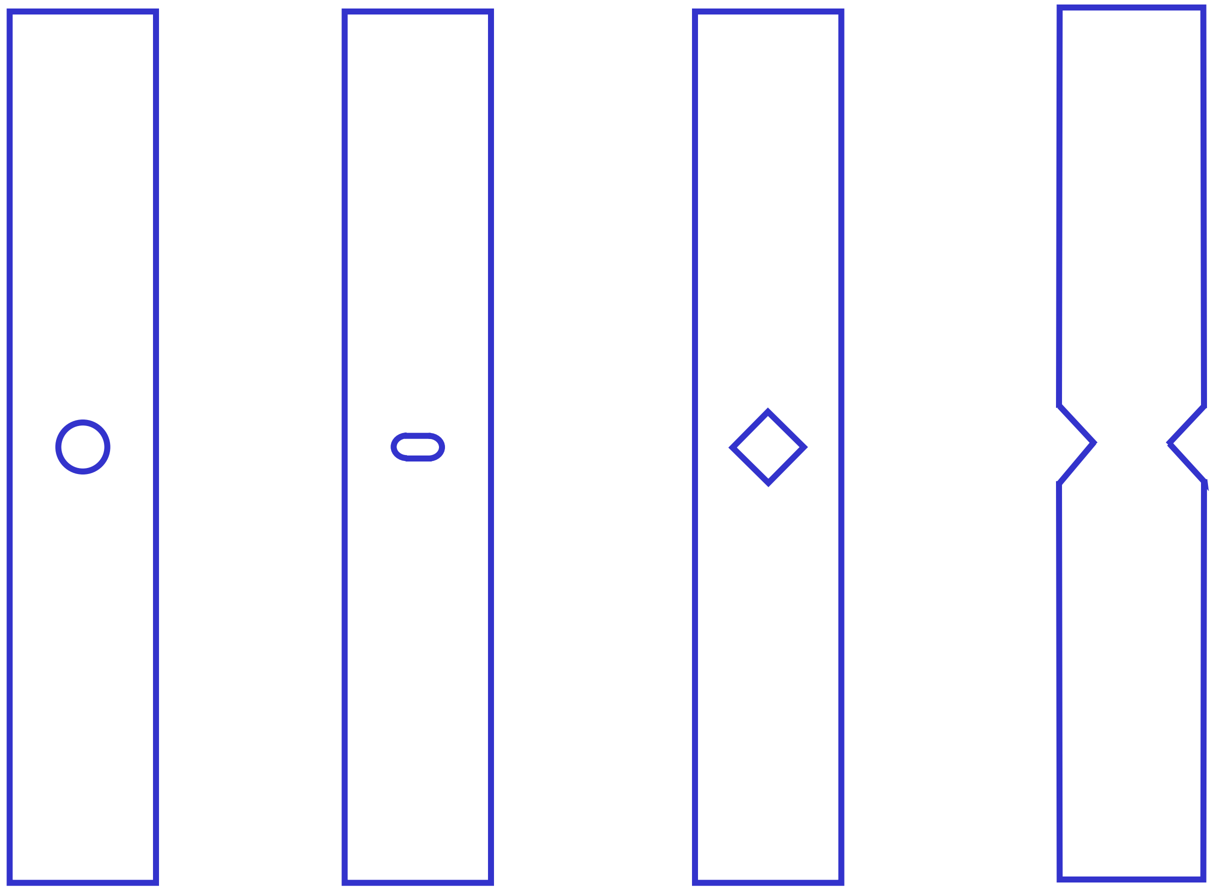
No effect when $\rho << \alpha$

Full effect when $\rho >> \alpha$

Pererson's Constant



Static Strength



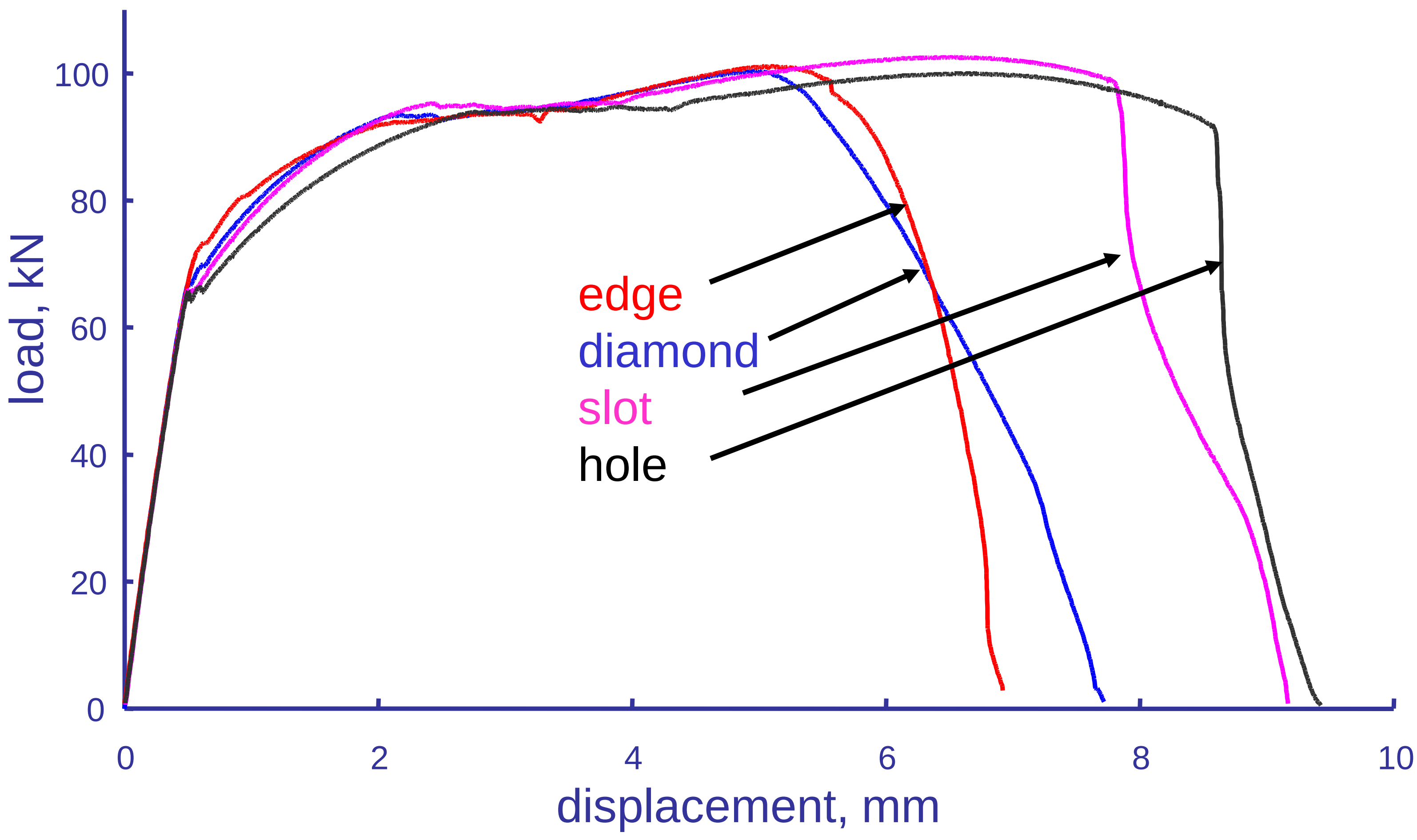
hole $K_t = 2.5$

slot $K_t = 5$

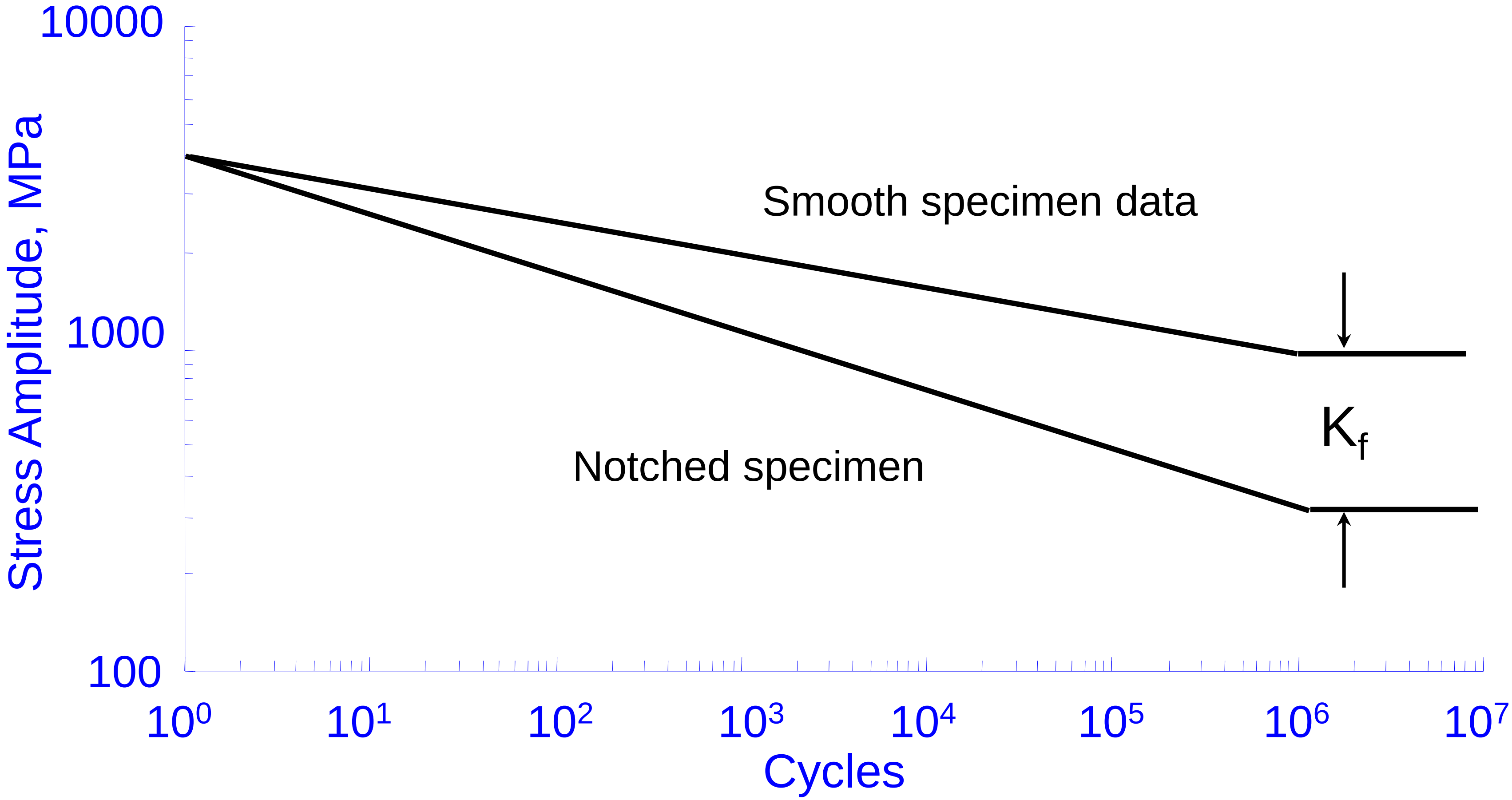
diamond $K_t = 20$

edge $K_t = 20$

1018 Steel Test Data



Notched SN Curve



Stress concentrations are not very important at short lives

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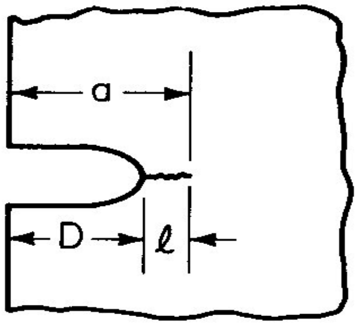
Smith - Miller



Long crack

$$l > 0.13\sqrt{D\rho}$$

$$K = \sigma\sqrt{\pi a}$$

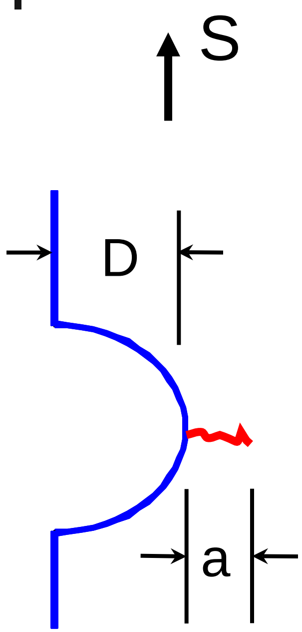
Short crack

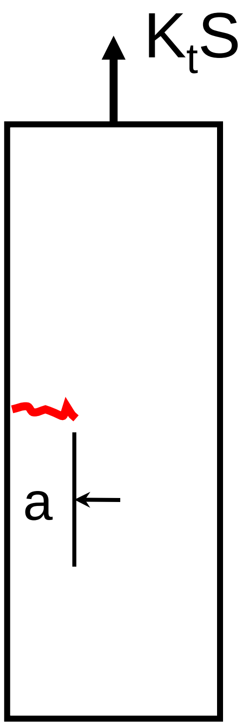
$$l < 0.13\sqrt{D\rho}$$

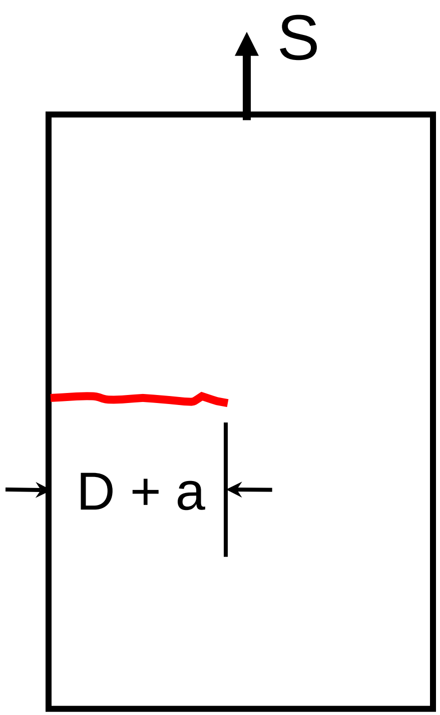
$$K = \left[1 + 7.69\sqrt{\frac{D}{\rho}}\right]\sigma\sqrt{\pi l}$$

$K_{fatigue}$

Cracks at Notches







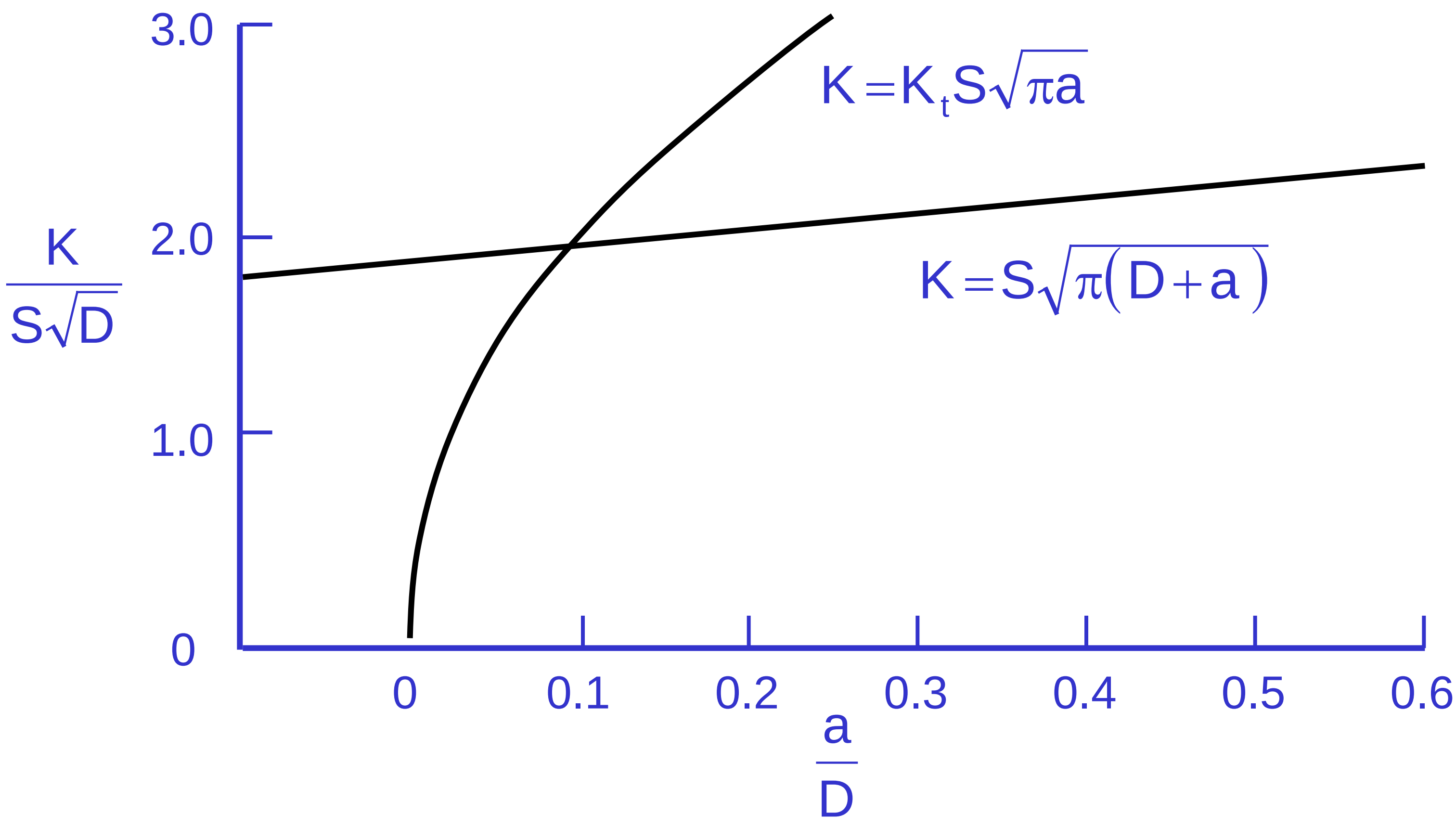
a << D

$$K = K_t S \sqrt{\pi a}$$

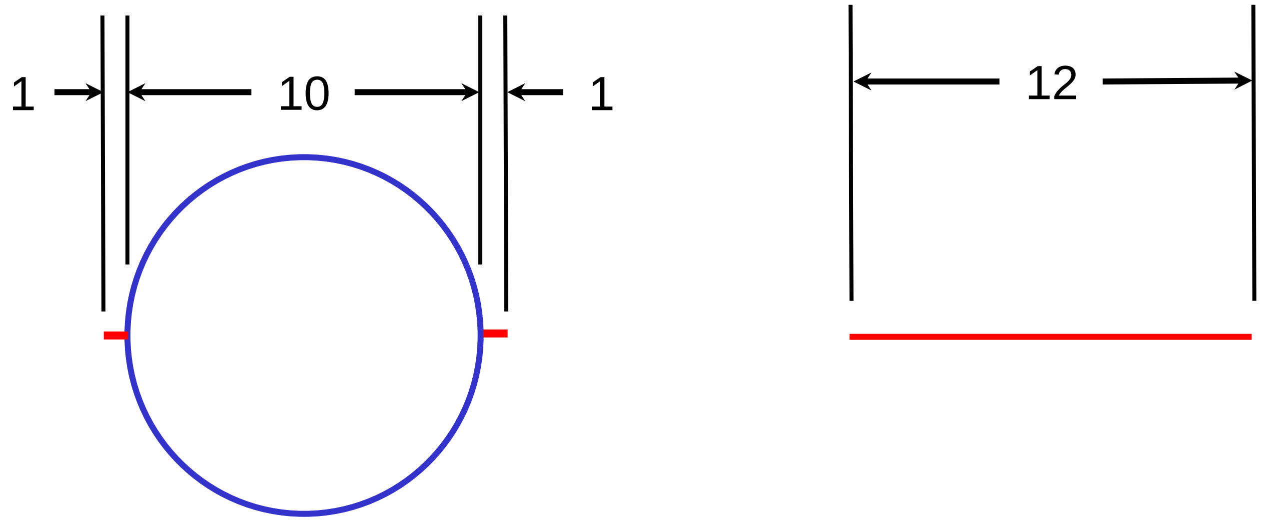
a >> D

$$K = S \sqrt{\pi (D + a)}$$

Stress Intensity Factors



Cracks at Holes



Once a crack reaches 10% of the hole radius, it behaves as if the hole was part of the crack

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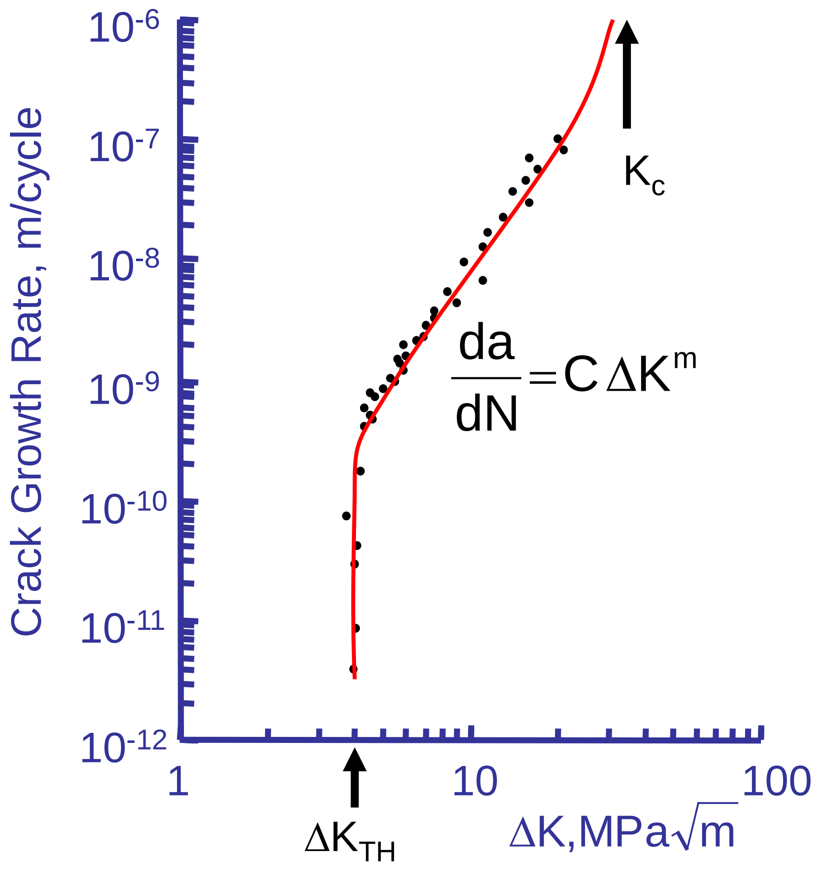
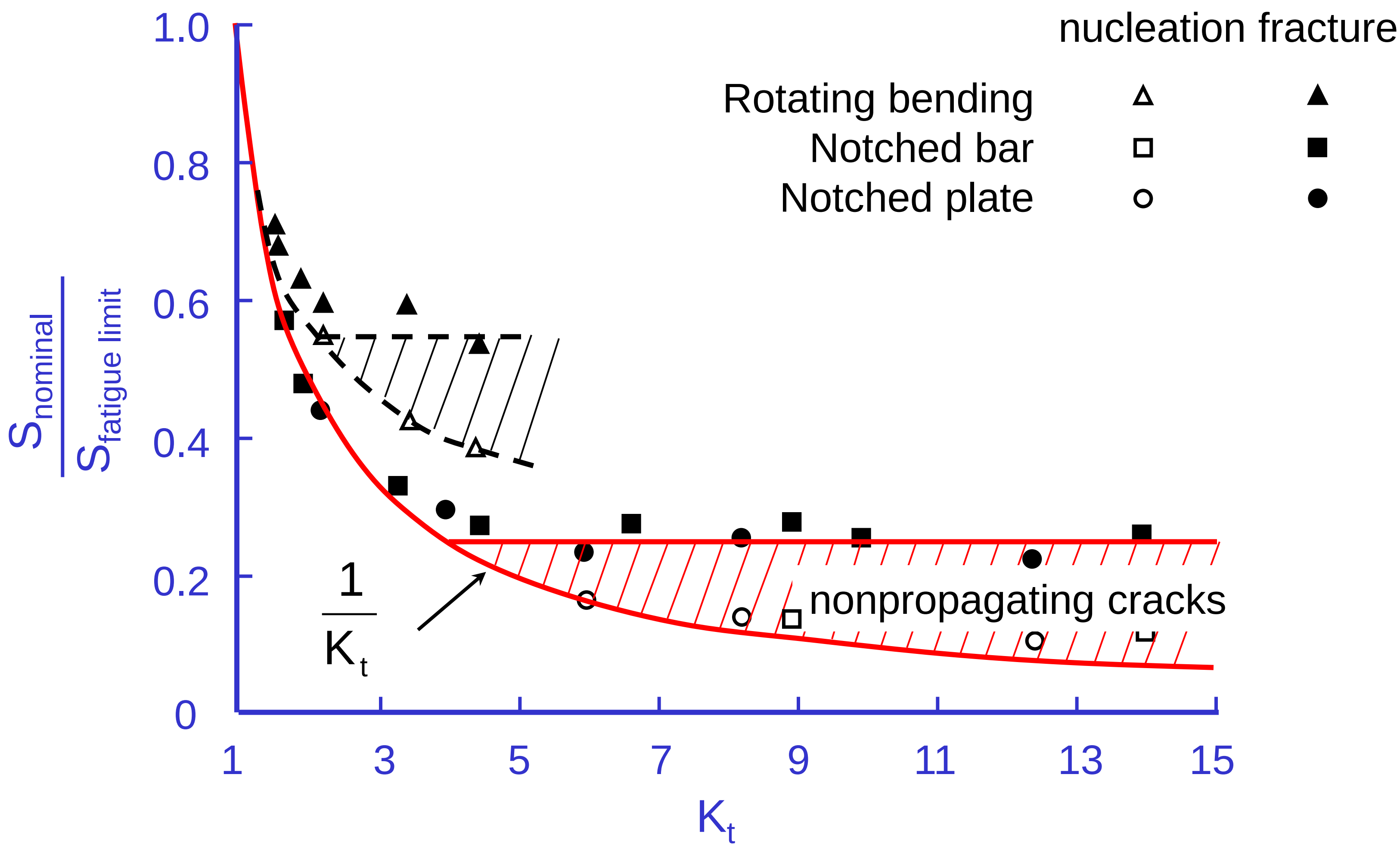
Crack Growth Data



Nonpropagating cracks

$$\Delta K_{TH} > \Delta\sigma 1.12 \frac{2}{\pi}\sqrt{\pi a}$$

Frost Data



Frost, "A Relation Between the Critical Alternating Propagation Stress and Crack Length for Mild Steel" Proceedings of the Institute for Mechanical Engineers, Vol. 173, No. 35, 1959, 811-836

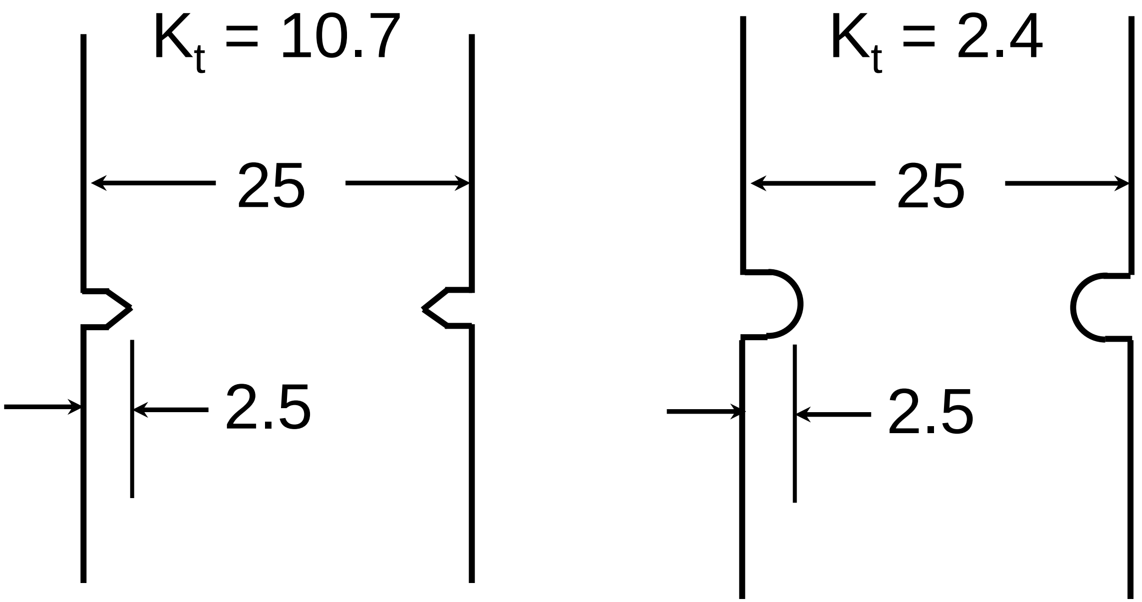
Significance

For $K_t > 4$, the notch acts like a crack with a depth D

$$S_{fl} = \frac{\Delta K_{th}}{\sqrt{\pi D}}$$

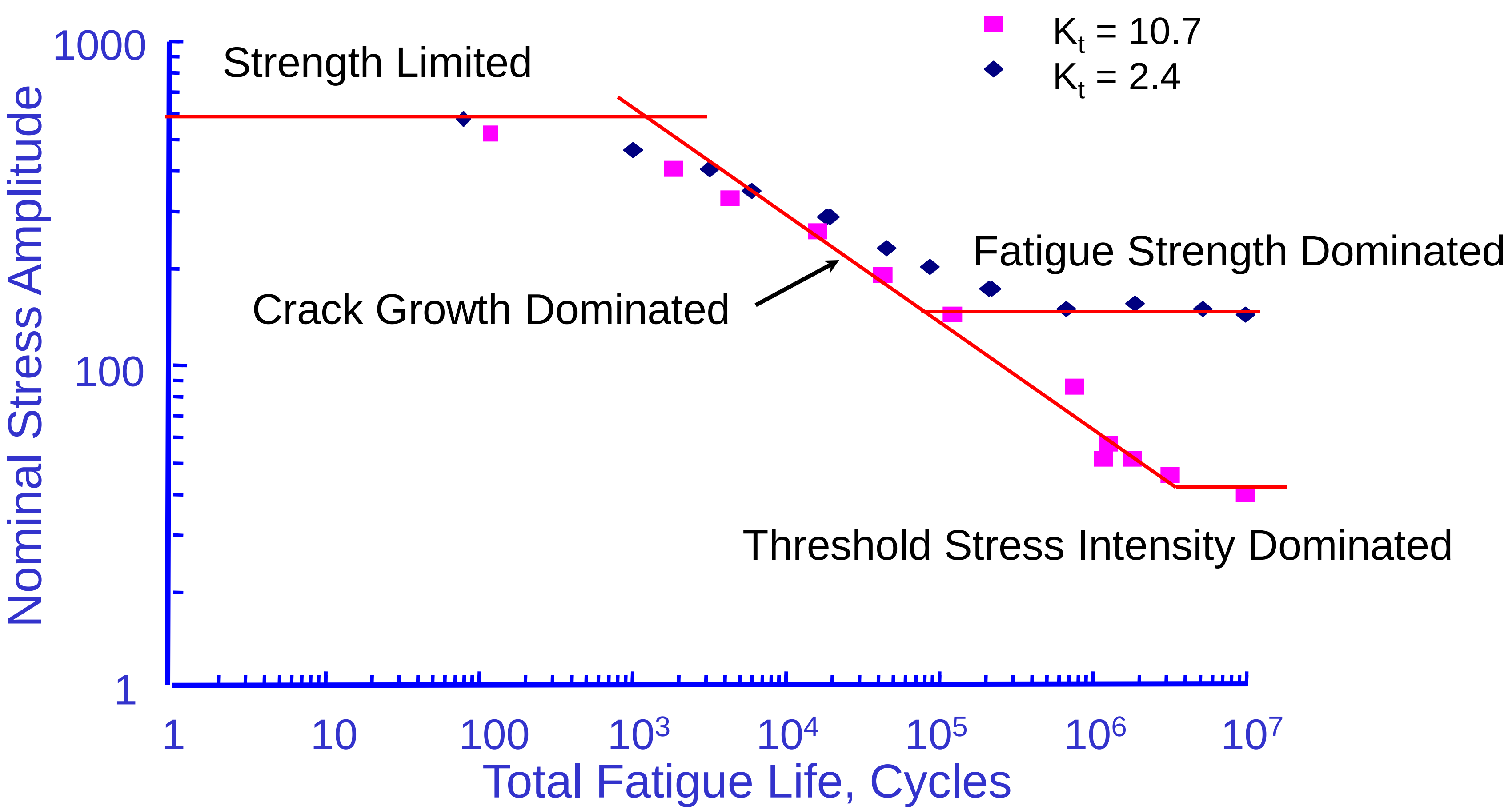
K_t does not play a role for sharp notches !

Specimens with Similar Geometry



Ultimate Strength 780 MPa
Yield Strength 660 MPa

Test Results



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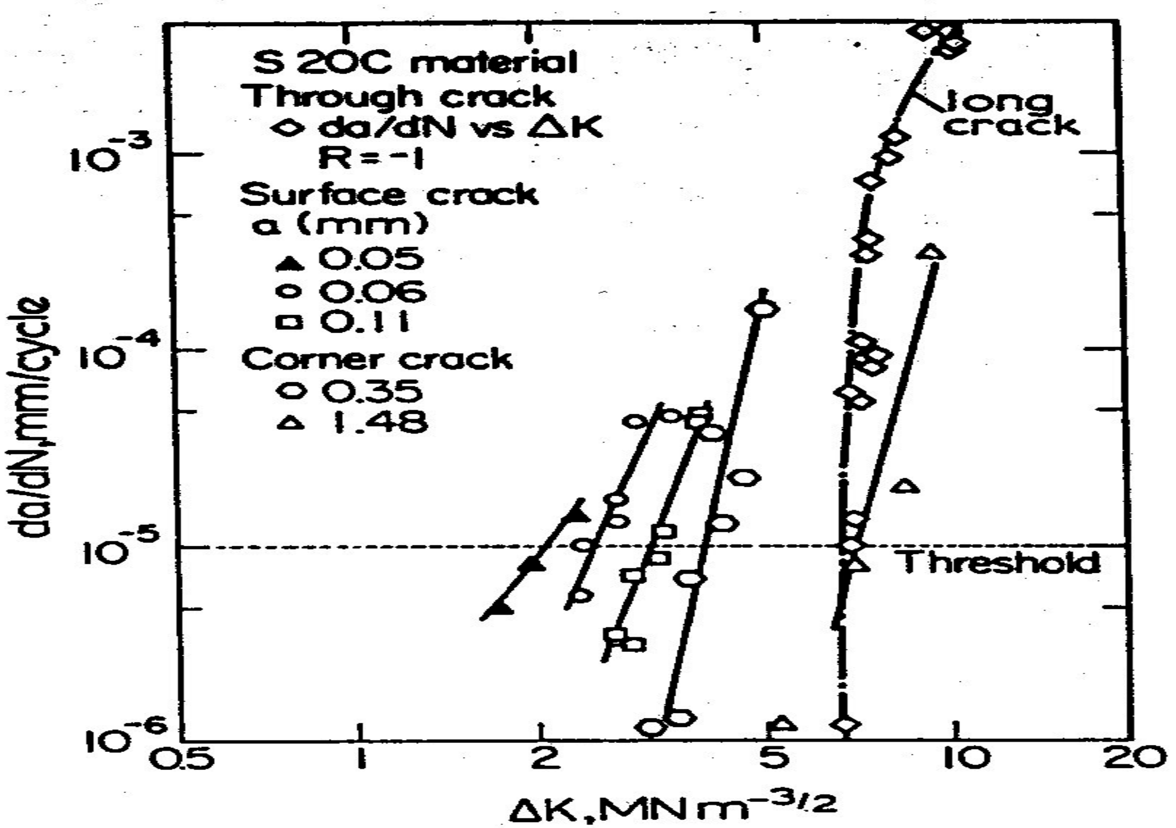
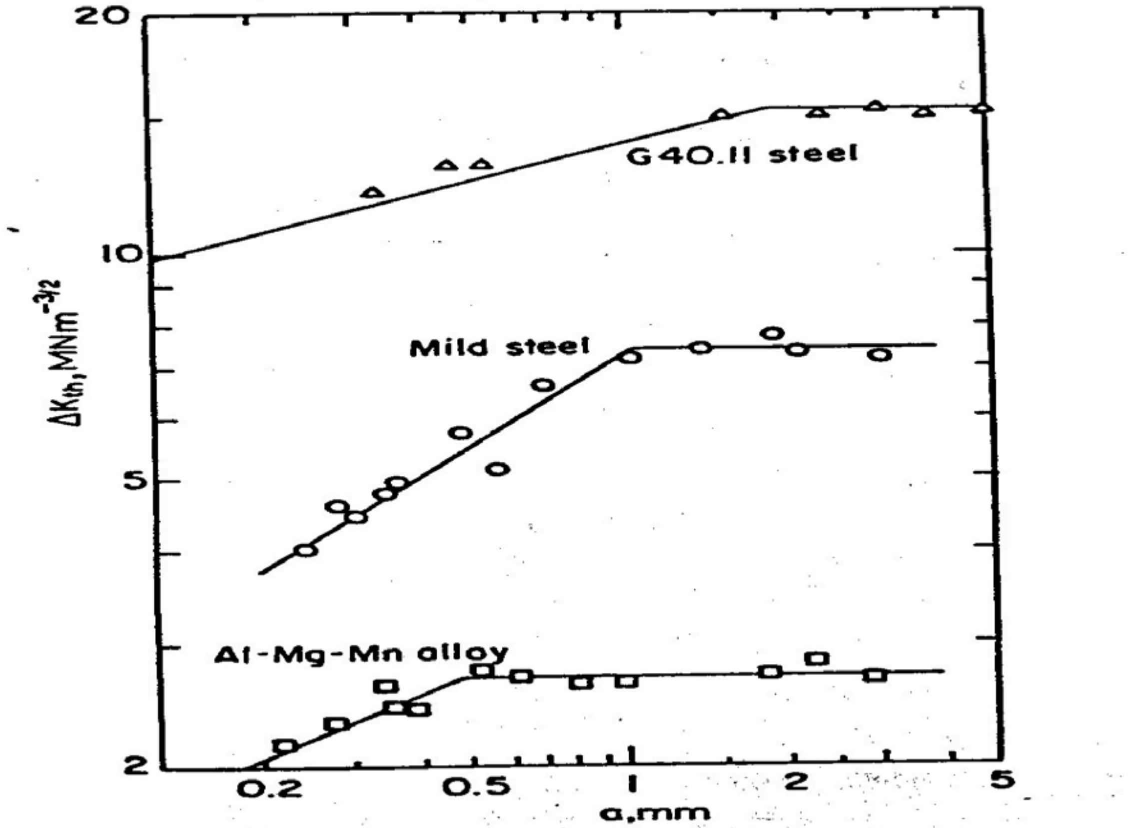
Small Crack Growth



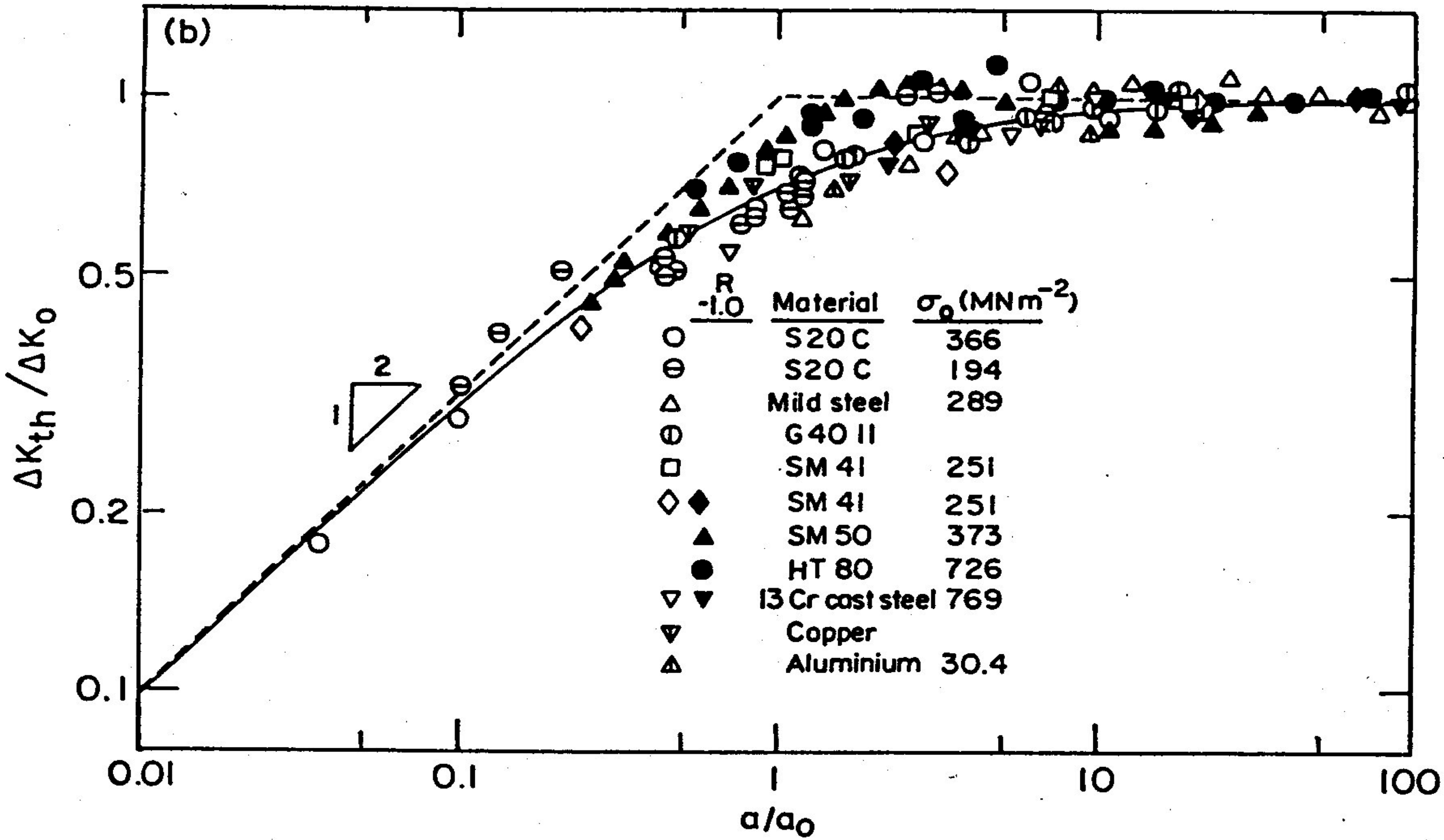
12 Difference in propagation rates da/dN of short and long fatigue cracks as function of stress intensity factor range ΔK for 3%Si iron of yield strength $\sigma_0 = 431$ MNm^{-2} (Ref. 70)

Threshold

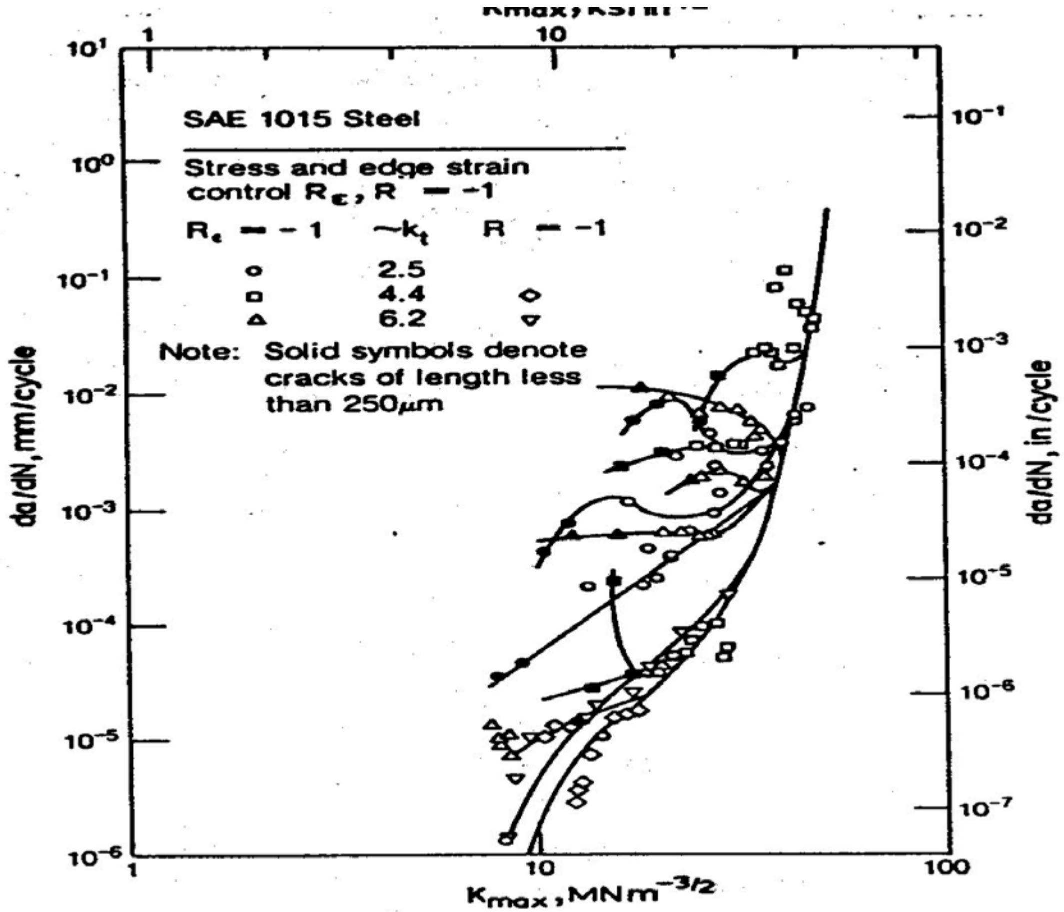


16 Variation of threshold stress intensity range ΔK_{th} with short crack length a in G40.11 austenitic 0.45%C steel, σ_o = 550 MNm^{-2}, 0.035%C mild steel, σ_o = 242 MNm^{-2}, and Al—Zn—Mg alloy, σ_o = 180 MNm^{-2} (Ref. 69)

Normalized Thresholds

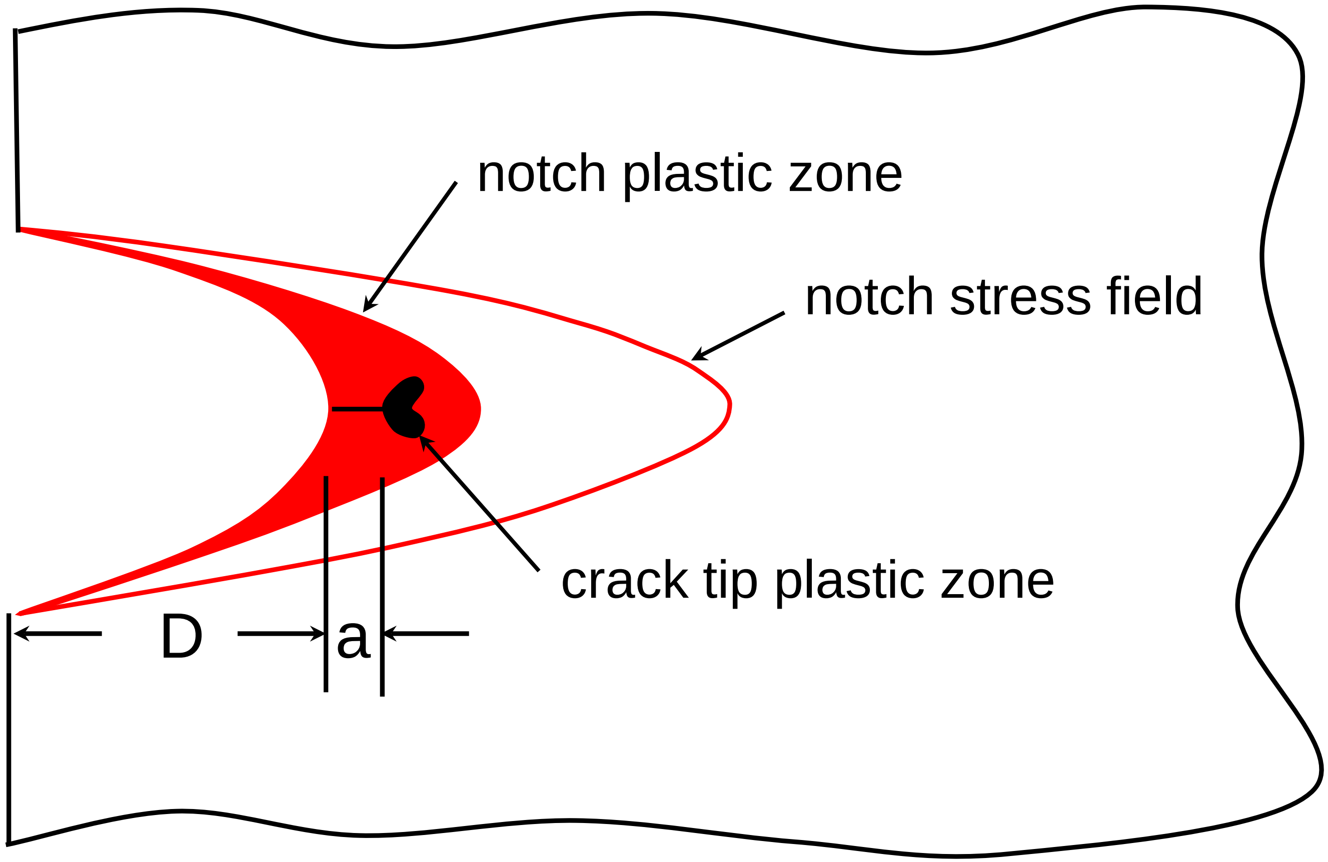


Growth from Notches

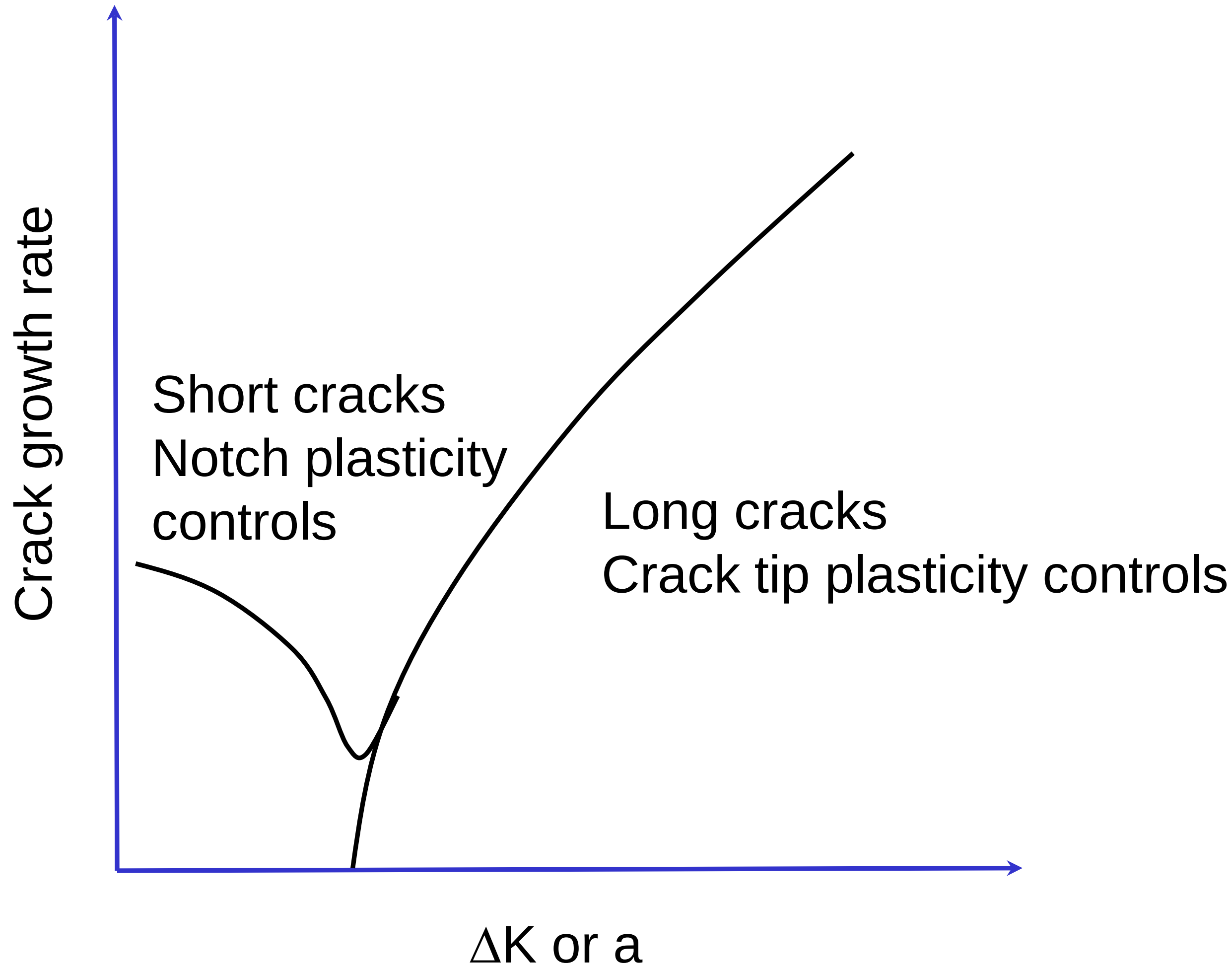


24 Propagation rate da/dN of cracks emanating from notches as function of maximum stress intensity factor K_{max} in 0.15%C mild steel; k_t is theoretical elastic stress concentration factor, R stress ratio, and R_ϵ edge strain ratio[110]

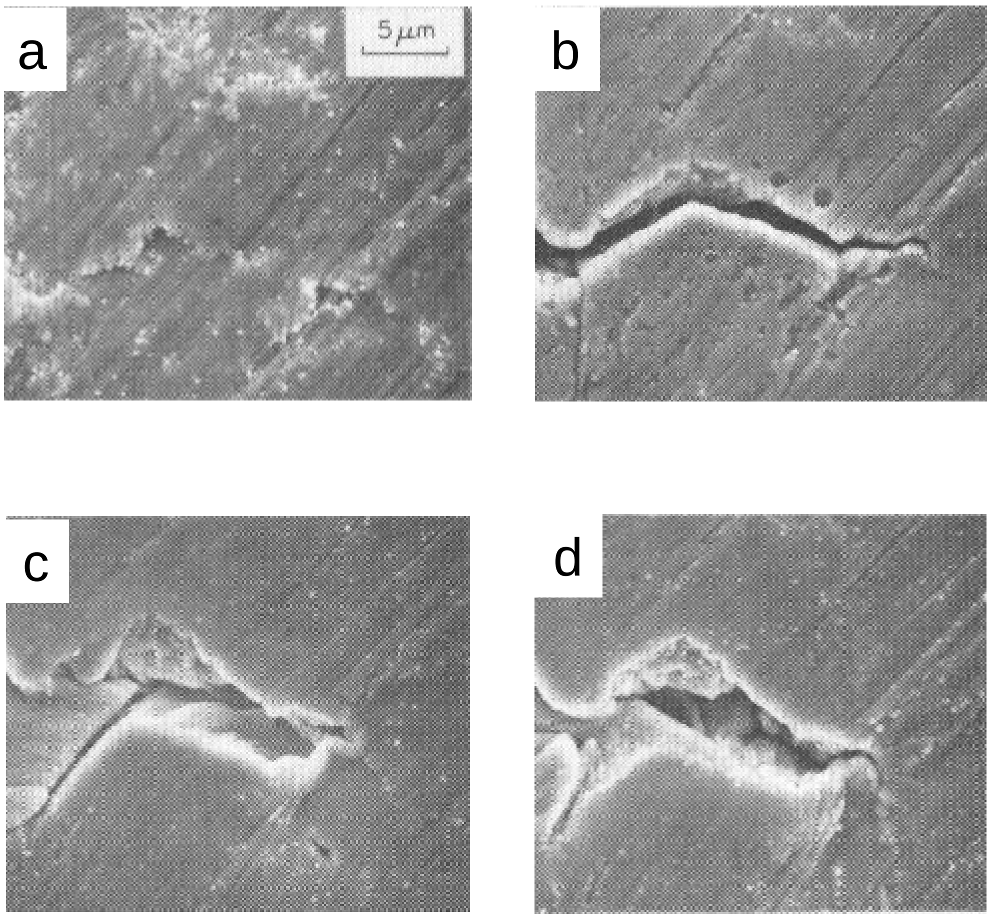
Cracks at Notches

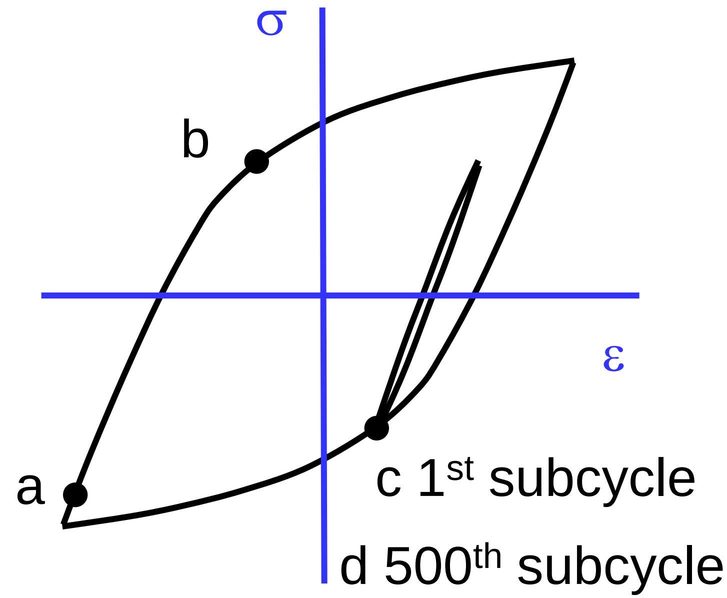


Crack Growth



Closure Observations



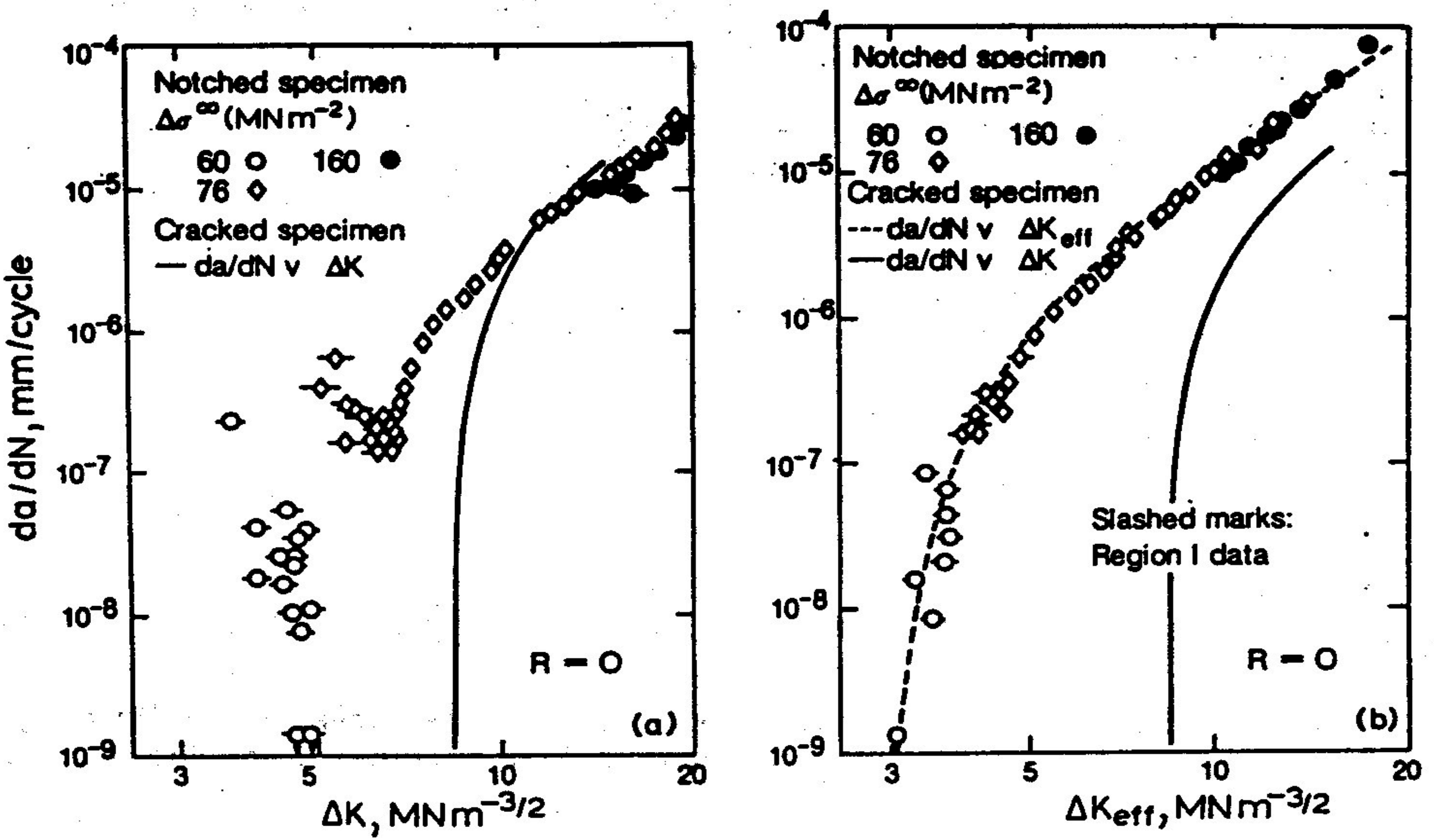


1026 steel

$\Delta\varepsilon_1/2 = 0.005$

$\Delta\varepsilon_2/2 = 0.001$

Closure Correlation



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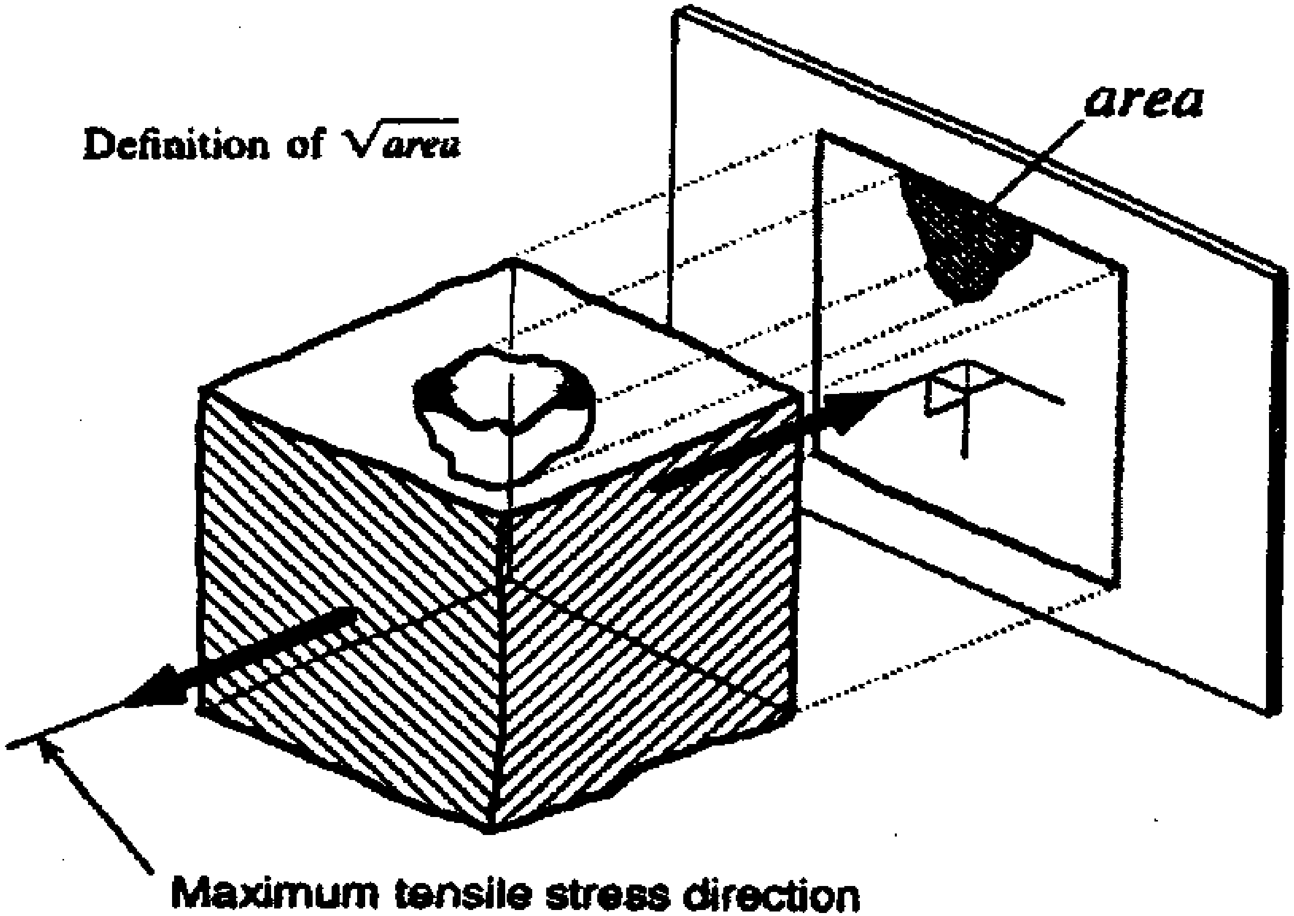
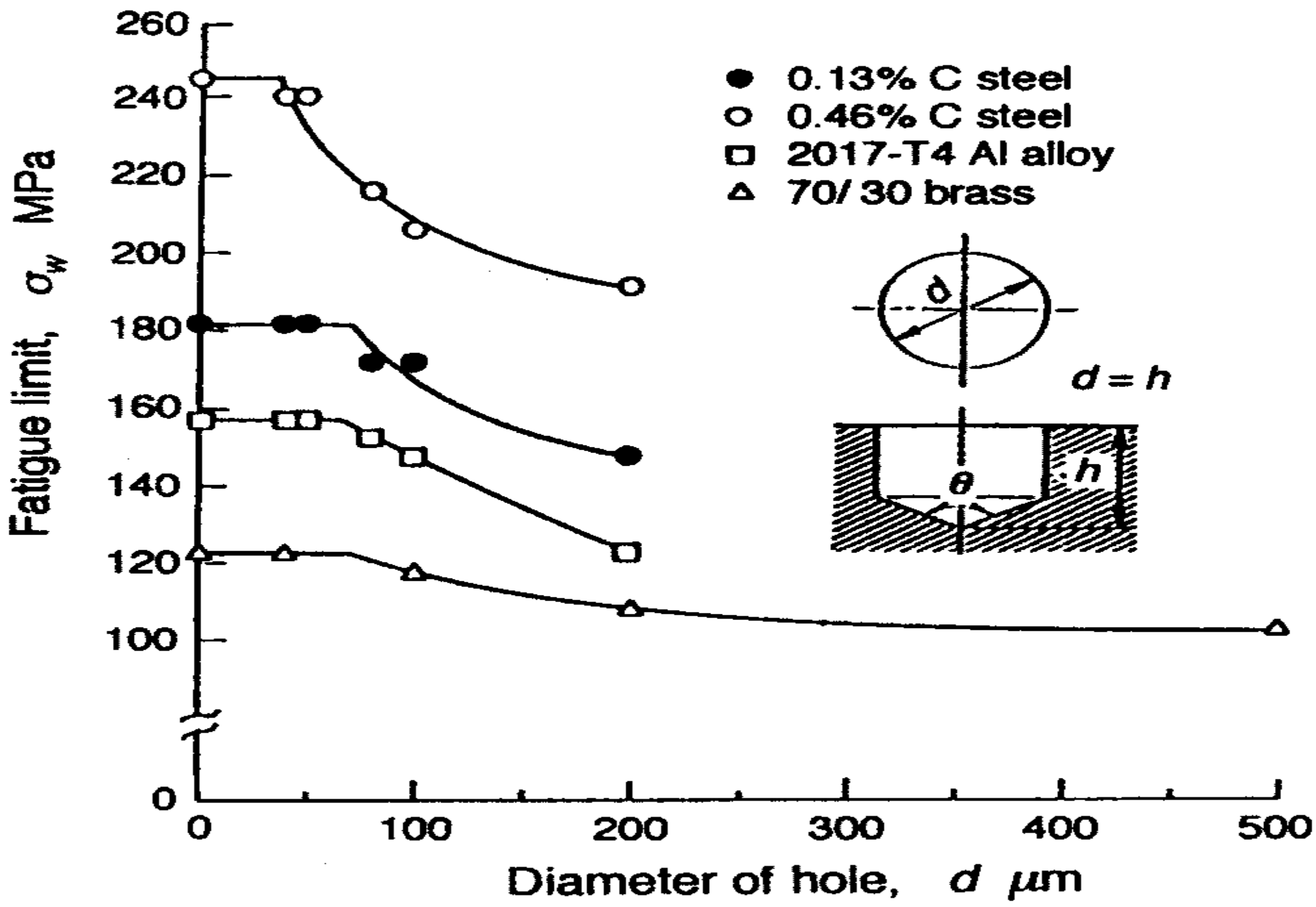
$\sqrt{area}$

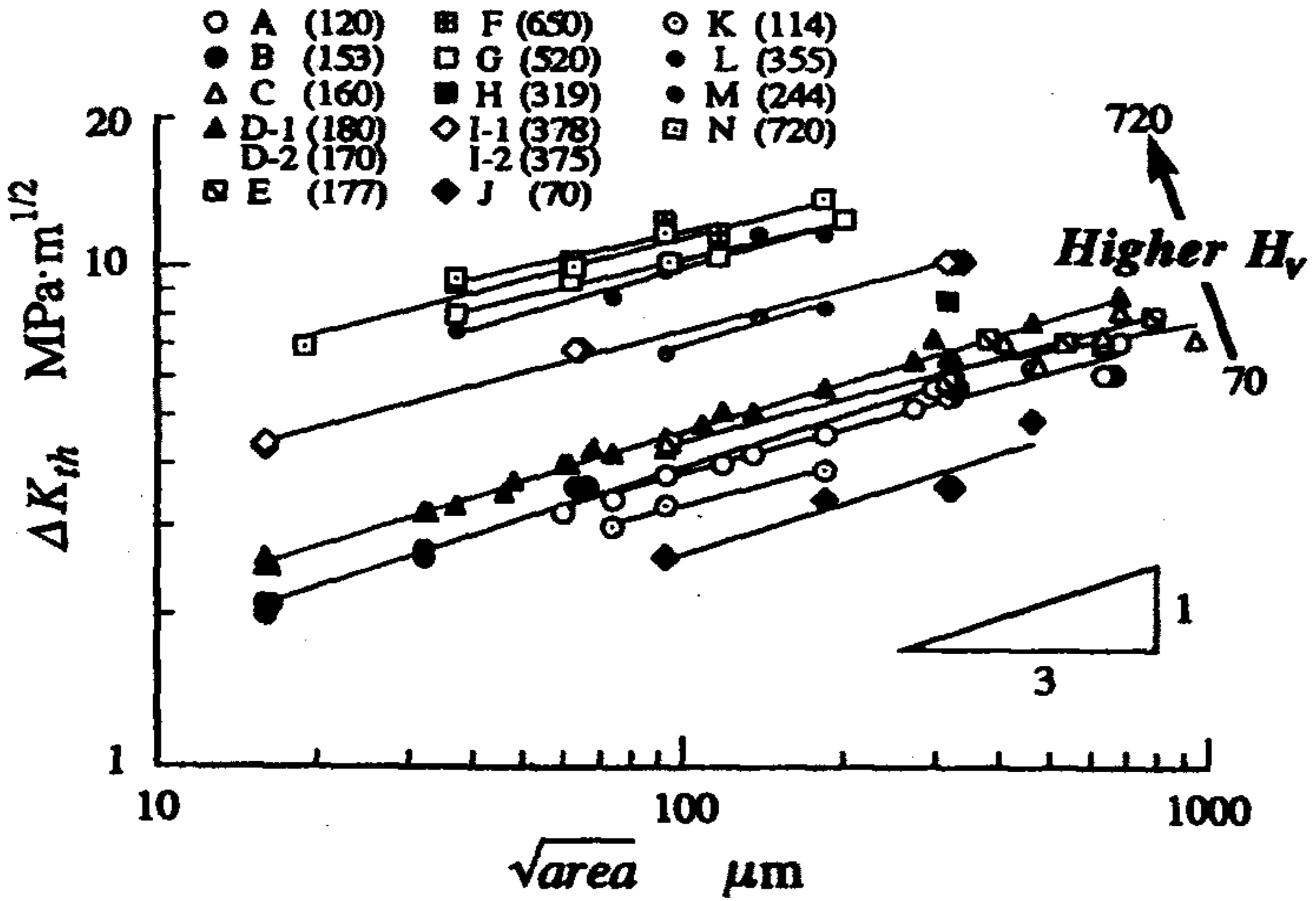


Definition of $\sqrt{area}$
area
Maximum tensile stress direction

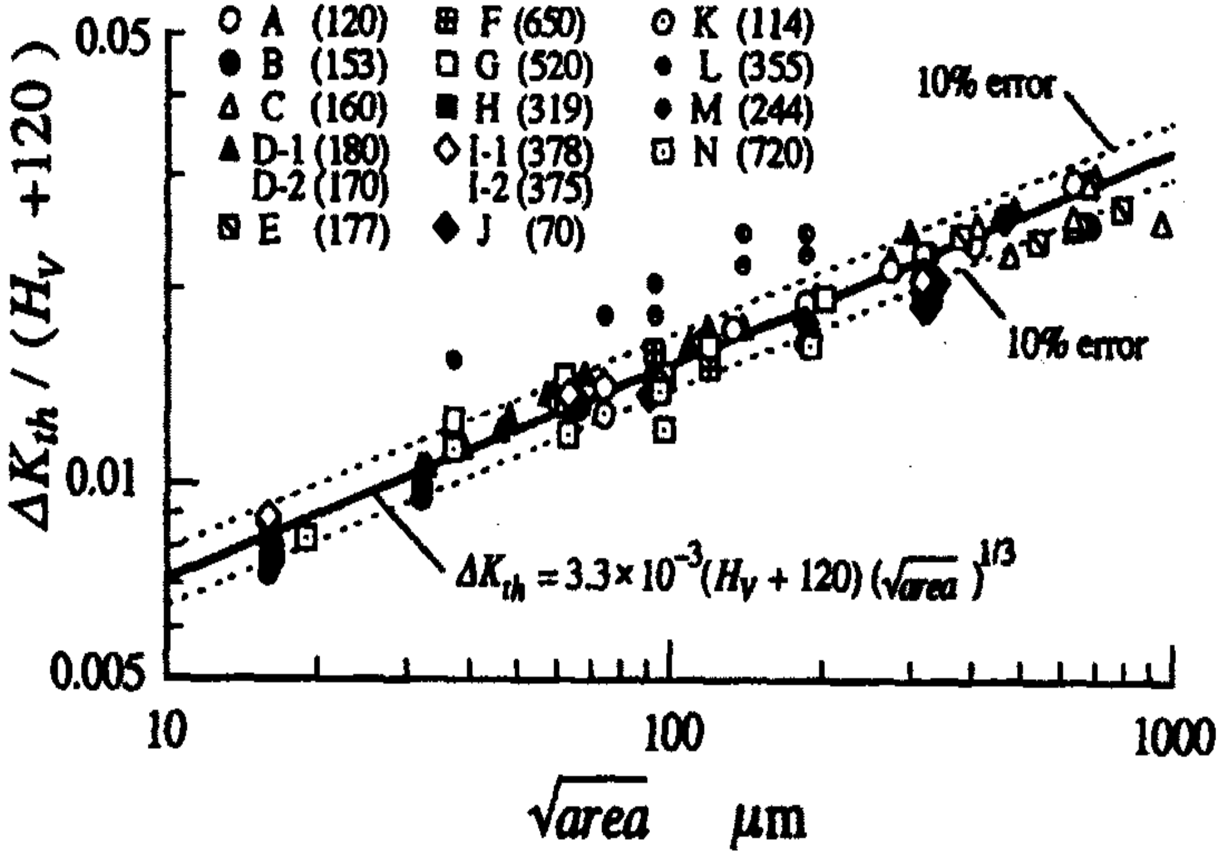
Small Notches



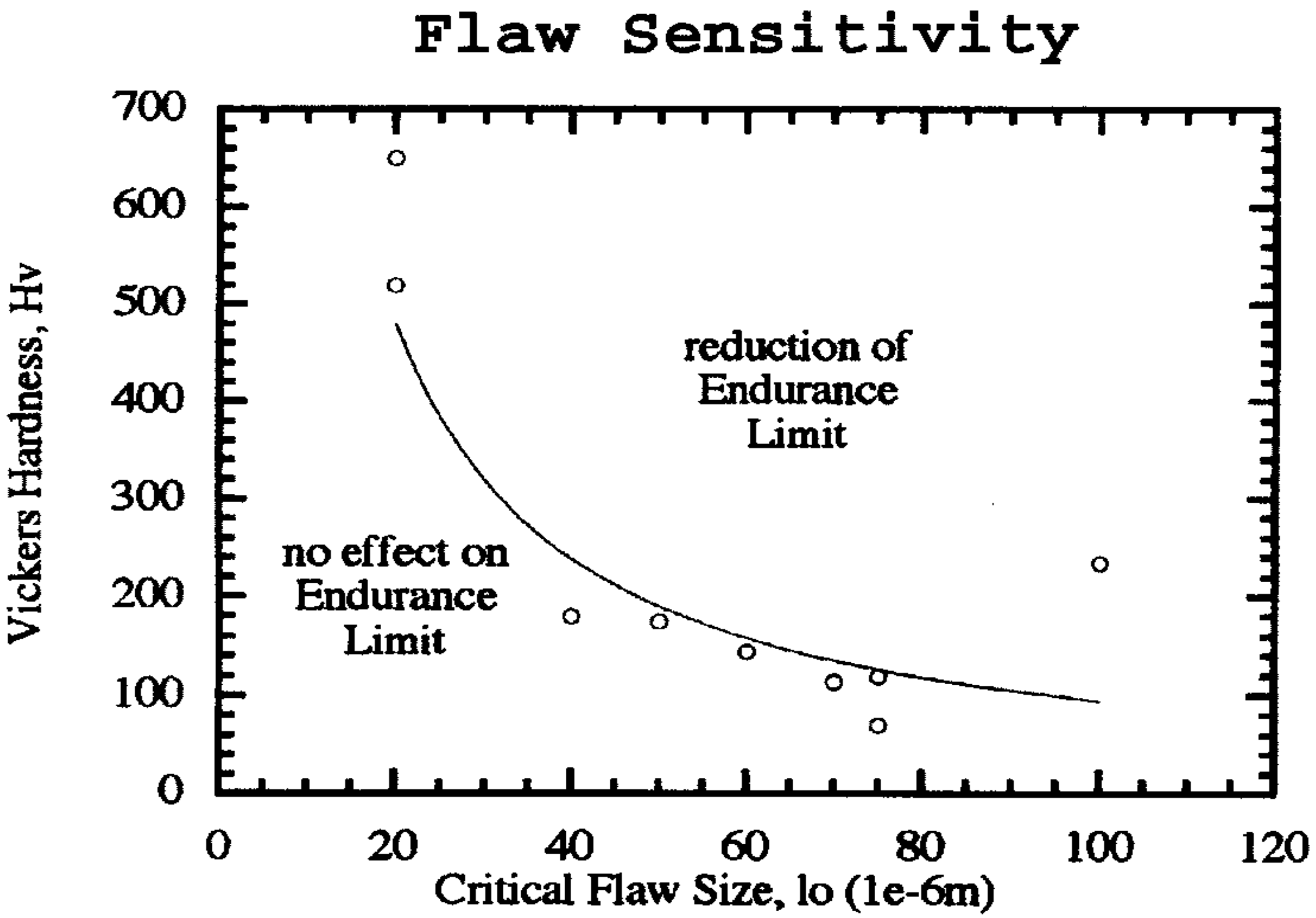
Threshold Stress Intensity



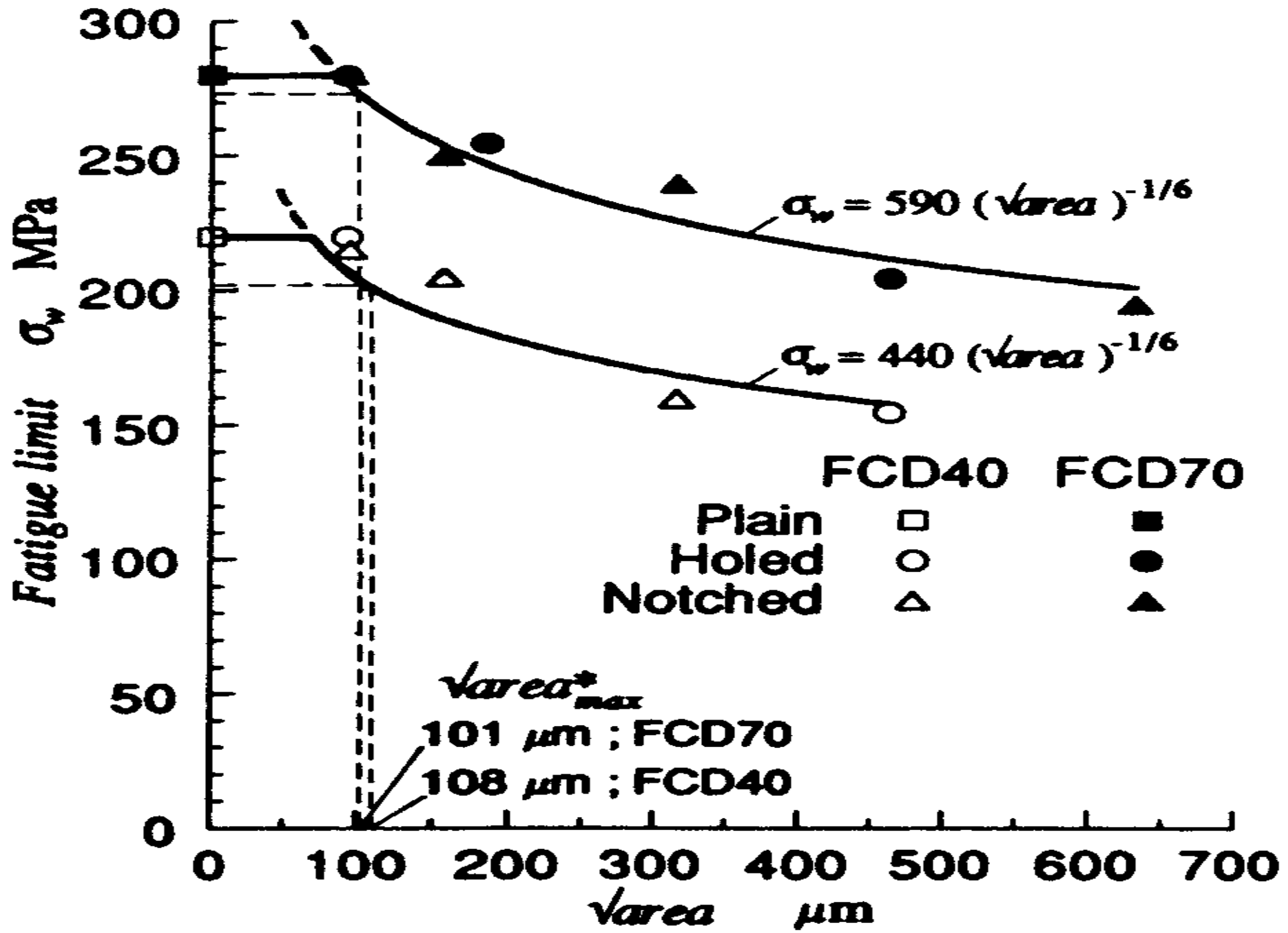
Hardness Corelation



Flaw Sensitivity



Fatigue Limit



Stress Concentration